DOWNLOAD PDF THE LOGIC OF PARADOX

Chapter 1 : Liar Paradox (Stanford Encyclopedia of Philosophy)

This is a list of paradoxes, grouped theinnatdunvilla.com grouping is approximate, as paradoxes may fit into more than one category. This list collects only scenarios that have been called a paradox by at least one source and have their own article.

The following list presents eight influential philosophical puzzles and paradoxes dating from ancient times to the present. Take a look and be perplexed. On the other hand, if what she tells you is false, then she is not lying, in which case what she tells you is true. It is attributed to the ancient Greek seer Epimenides fl. The paradox is important in part because it creates severe difficulties for logically rigorous theories of truth; it was not adequately addressed which is not to say solved until the 20th century. In the 5th century BCE, Zeno of Elea devised a number of paradoxes designed to show that reality is single there is only one thing and motionless, as his friend Parmenides had claimed. The paradoxes take the form of arguments in which the assumption of plurality the existence of more than one thing or motion are shown to lead to contradictions or absurdity. Here are two of the arguments: A Suppose that reality is plural. Then the number of things there are is only as many as the number of things there are the number of things there are is neither more nor less than the number of things there are. If the number of things there are is only as many as the number of things there are, then the number of things there are is finite. B Suppose that reality is plural. Then there are at least two distinct things. Two things can be distinct only if there is a third thing between them even if it is only air. It follows that there is a third thing that is distinct from the other two. But if the third thing is distinct, then there must be a fourth thing between it and the second or first thing. And so on to infinity. C Therefore, if reality is plural, it is finite and not finite, infinite and not infinite, a contradiction. Suppose that there is motion. Suppose in particular that Achilles and a tortoise are moving around a track in a foot race, in which the tortoise has been given a modest lead. Naturally, Achilles is running faster than the tortoise. If Achilles is at point A and the tortoise at point B, then in order to catch the tortoise Achilles will have to traverse the interval AB. But in the time it takes Achilles to arrive at point B, the tortoise will have moved on however slowly to point C. Then in order to catch the tortoise, Achilles will have to traverse the interval BC. But in the time it takes him to arrive at point C, the tortoise will have moved on to point D, and so on for an infinite number of intervals. It follows that Achilles can never catch the tortoise, which is absurd. Consider a single grain of rice, which is not a heap. Adding one grain of rice to it will not create a heap. Likewise adding one grain of rice to two grains or three grains or four grains. Similarly, if N grains does constitute a heap, then N-1 grains also constitutes a heap. It follows that one can never create a heap of rice from something that is not a heap of rice by adding one grain at a time. But that is absurd. Imagine a hungry donkey who is placed between two equidistant and identical bales of hay. Assume that the surrounding environments on both sides are also identical. The donkey cannot choose between the two bales and so dies of hunger, which is absurd. Yet the donkey will not starve. Leibniz, for what it is worth, vehemently rejected the paradox, claiming that it was unrealistic. The students begin to speculate about when it might occur, until one of them announces that there is no reason to worry, because a surprise test is impossible. The test cannot be given on Friday, she says, because by the end of the day on Thursday we would know that the test must be given the next day. Nor can the test be given on Thursday, she continues, because, given that we know that the test cannot be given on Friday, by the end of the day on Wednesday we would know that the test must be given the next day. And likewise for Wednesday, Tuesday, and Monday. The students spend a restful weekend not studying for the test, and they are all surprised when it is given on Wednesday. How could this happen? There are various versions of the paradox; one of them, called the Hangman, concerns a condemned prisoner who is clever but ultimately overconfident. The implications of the paradox are as yet unclear, and there is virtually no agreement about how it should be solved. You buy a lottery ticket, for no good reason. Indeed, you know that the chance that your ticket will win is at least 10 million to one, since at least 10 million tickets have been sold, as you learn later on the evening news, before the drawing assume that the lottery is fair and that a winning ticket exists. In general, for each ticket sold in the lottery, you are justified in believing: But, of course, you know that one ticket will win.

How can that be? The lottery constitutes an apparent counterexample to one version of a principle known as the deductive closure of justification: If one is justified in believing P and justified in believing Q, then one is justified in believing any proposition that follows deductively necessarily from P and Q. For example, if I am justified in believing that my lottery ticket is in the envelope because I put it there , and if I am justified in believing that the envelope is in the paper shredder because I put it there , then I am justified in believing that my lottery ticket is in the paper shredder. Since its introduction in the early s, the lottery paradox has provoked much discussion of possible alternatives to the closure principle, as well as new theories of knowledge and belief that would retain the principle while avoiding its paradoxical consequences. Socrates and Meno are engaged in a conversation about the nature of virtue. Meno offers a series of suggestions, each of which Socrates shows to be inadequate. Socrates himself professes not to know what virtue is. How then, asks Meno, would you recognize it, if you ever encounter it? It seems to follow that no one ever learns anything by asking questions, which is implausible, if not absurd. As proof he shows how a slave boy can be prompted to solve geometrical problems, though he has never had instruction in geometry. In this case, the answer, originally proposed by Noam Chomsky in the s, is that the basic elements of the grammars of all human languages are innate, ultimately a genetic endowment reflecting the cognitive evolution of the human species. It begins to rain outside. Why, then, is the second sentence absurd?

Chapter 2 : 8 Philosophical Puzzles and Paradoxes | theinnatdunvilla.com

Hyper-Contradictions, Generalized Truth Values and Logics of Truth and Falsehood. Yaroslav Shramko & Heinrich Wansing - - Journal of Logic, Language and Information 15 (4)

February 10, iStock A paradox is a statement or problem that either appears to produce two entirely contradictory yet possible outcomes, or provides proof for something that goes against what we intuitively expect. Paradoxes have been a central part of philosophical thinking for centuries, and are always ready to challenge our interpretation of otherwise simple situations, turning what we might think to be true on its head and presenting us with provably plausible situations that are in fact just as provably impossible. It begins with the great hero Achilles challenging a tortoise to a footrace. To keep things fair, he agrees to give the tortoise a head start of, say, m. When the race begins, Achilles unsurprisingly starts running at a speed much faster than the tortoise, so that by the time he has reached the m mark, the tortoise has only walked 50m further than him. But by the time Achilles has reached the m mark, the tortoise has walked another 5m. And by the time he has reached the m mark, the tortoise has walked another 0. This process continues again and again over an infinite series of smaller and smaller distances, with the tortoise always moving forwards while Achilles always plays catch up. Logically, this seems to prove that Achilles can never overtake the tortoiseâ€"whenever he reaches somewhere the tortoise has been, he will always have some distance still left to go no matter how small it might be. Except, of course, we know intuitively that he can overtake the tortoise. Imagine that a time traveller buys a copy of Hamlet from a bookstore, travels back in time to Elizabethan London, and hands the book to Shakespeare, who then copies it out and claims it as his own work. Over the centuries that follow, Hamlet is reprinted and reproduced countless times until finally a copy of it ends up back in the same original bookstore, where the time traveller finds it, buys it, and takes it back to Shakespeare. Who, then, wrote Hamlet? What then is the probability that the other child is a boy? In a two-child family, however, there are actually four possible combinations of children: We already know that one of the children is a boy, meaning we can eliminate the combination FF, but that leaves us with three equally possible combinations of children in which at least one is a boyâ€"namely MM, MF, and FM. Trying to assign any truth to either Statement A or B, however, leads to a paradox: Oppositely, if A is false then B must be false too, which must ultimately make A true. An even more complicated variation of a liar paradox is the next entry on our list. His mother pleads with the crocodile to return him, to which the crocodile replies that he will only return the boy safely if the mother can guess correctly whether or not he will indeed return the boy. There is no problem if the mother guesses that the crocodile will return himâ€"if she is right, he is returned; if she is wrong, the crocodile keeps him. If she answers that the crocodile will not return him, however, we end up with a paradox: On the other hand, if she is wrong and the crocodile actually did intend to return the boy, the crocodile must then keep him even though he intended not to, thereby also breaking his word. The Crocodile Paradox is such an ancient and enduring logic problem that in the Middle Ages the word "crocodilite" came to be used to refer to any similarly brain-twisting dilemma where you admit something that is later used against you, while "crocodility" is an equally ancient word for captious or fallacious reasoning 6. And before that a sixteenth of the way there, and then a thirty-second of the way there, a sixty-fourth of the way there, and so on. So, for that instant in time, the arrow must be stationary. But because all time is comprised entirely of instantsâ€"in every one of which the arrow must also be stationaryâ€"then the arrow must in fact be stationary the entire time. On the one hand, he proposed, there are square numbersâ€"like 1, 4, 9, 16, 25, 36, and so on. On the other, there are those numbers that are not squaresâ€"like 2, 3, 5, 6, 7, 8, 10, and so on. Put these two groups together, and surely there have to be more numbers in general than there are just square numbersâ€"or, to put it another way, the total number of square numbers must be less than the total number of square and non-square numbers together. However, because every positive number has to have a corresponding square and every square number has to have a positive number as its square root, there cannot possibly be more of one than the other. In his discussion of his paradox, Galileo was left with no alternative than to conclude that numerical concepts like more, less, or fewer can only be applied to finite sets of numbers, and as there are an infinite number of square and non-square numbers, these concepts simply cannot be used in this context. But when he returns to them the day after, he finds his lb sack now weighs just 50 lbs. How can this be true? Although not a true paradox in the strictest sense, the counterintuitive Potato Paradox is a famous example of what is known as a veridical paradox, in which a basic theory is taken to a logical but apparently absurd conclusion. But by extension, whenever we see anything that is not black, like an apple, this too must be taken as evidence supporting the second statementâ€"after all, an apple is not black, and nor is it a raven. The paradox here is that Hempel has apparently proved that seeing an apple provides us with evidence, no matter how unrelated it may seem, that ravens are black. Just how much information can one statement actually imply anyway?

Chapter 3 : Zenoâ€™s Paradox of the Tortoise and Achilles

Reprinted in English in Logic Semantics and Metamathematics, Oxford U.P., Google Scholar TarskiA., 'The Semantic Conception of Truth and the Foundations of Semantics', Philosophy and Phenomological Research 4 (),

The second law of thermodynamics seems to be violated by a cleverly operated trapdoor. Hot water can, under certain conditions, freeze faster than cold water, even though it must pass the lower temperature on the way to freezing. Biology[edit] Antarctic paradox: In some areas of the oceans, phytoplankton concentrations are low despite there apparently being sufficient nutrients. Genome size does not correlate with organismal complexity. For example, some unicellular organisms have genomes much larger than that of humans. Even a tiny fecundity advantage of one additional offspring would favor the evolution of semelparity. Despite their relatively small muscle mass, dolphins can swim at high speeds and obtain large accelerations. Exposure to small doses of toxins can have beneficial effects. Persistent female choice for particular male trait values should erode genetic variance in male traits and thereby remove the benefits of choice, yet choice persists. When rising to stand from a sitting or squatting position, both the hamstrings and quadriceps contract at the same time, despite their being antagonists to each other. Increasing the food available to an ecosystem may lead to instability, and even to extinction. Paradox of the pesticides: Paradox of the plankton: Why are there so many different species of phytoplankton, even though competition for the same resources tends to reduce the number of species? An anomalous pattern of inheritance in the fragile X syndrome. When did the ancestors of birds live? Health and nutrition[edit] French paradox: The observation that the French suffer a relatively low incidence of coronary heart disease, despite having a diet relatively rich in saturated fats, which are assumed to be the leading dietary cause of such disease. The large amount of glycogen in the liver cannot be explained by its small glucose absorption. The finding that Hispanics in the United States tend to have substantially better health than the average population in spite of what their aggregate socio-economic indicators predict. The observation that Israelis suffer a relatively high incidence of coronary heart disease, despite having a diet very low in saturated fats, which are assumed to be the leading dietary cause of such disease. The amplitude of heart rate oscillations during meditation was significantly greater than in the pre-meditation control state and also in three non-meditation control groups [5] Mexican paradox: Mexican children tend to have higher birth weights than can be expected from their socio-economic status. Although the negative health consequences of obesity in the general population are well supported by the available evidence, health outcomes in certain subgroups seem to be improved at an increased BMI. Humans and other small-to-medium-sized mammals get cancer with high frequency, while larger mammals, like whales, do not. If cancer is essentially a negative outcome lottery at the cell level, and larger organisms have more cells, and thus more potentially cancerous cell divisions, one would expect larger organisms to be more predisposed to cancer. A pulsus paradoxus is an exaggerated decrease in systolic blood pressure during inspiration. It can indicate certain medical conditions in which there is reduced cardiac output, such as cardiac tamponade or constrictive pericarditis. Also known as the Pulse Paradox. Although the individual is more wakeful and aware of their surroundings, they are continuing to accrue sleep debt and thus, are actually exacerbating their sleep deprivation. Chemistry[edit] Faraday paradox electrochemistry: Diluted nitric acid will corrode steel, while concentrated nitric acid will not. The length of time that it takes for a protein chain to find its folded state is many orders of magnitude shorter than it would be if it freely searched all possible configurations. Exceptions to the principle that a small change in a molecule causes a small change in its chemical behavior are frequently profound. Time travel[edit] Bootstrap paradox , also ontological paradox Can a time traveler send himself information with no outside source? A billiard ball can be thrown into a wormhole in such a way that it would emerge in the past and knock its incoming past self away from the wormhole entrance, creating a variant of the grandfather paradox. A man travels back in time to discover the cause of a famous fire. While in the building where the fire started, he accidentally knocks over a kerosene lantern and causes a fire, the same fire that would inspire him, years later, to travel back in time. The bootstrap paradox is closely tied to this, in which, as a result of time travel, information or objects appear to have no beginning. What happens when a time traveler does things in the past that prevent him from doing them in the first place? You travel back in time and kill a famous person in history before they become famous; but if the person had never been famous, then he could not have been targeted as a famous person. Linguistics and artificial intelligence[edit] Bracketing paradox: Is a "historical linguist" a linguist who is historical, or someone who studies "historical linguistics"? How can a language both enable communication and block communication? Logical thought is hard for humans and easy for computers, but picking a screw from a box of screws is an unsolved problem. In transformational linguistics, there are pairs of sentences in which the sentence without movement is ungrammatical while the sentence with movement is not. In automated handwriting recognition, a cursively written word cannot be recognized without being segmented and cannot be segmented without being recognized. Philosophy[edit] Paradox of analysis: It seems that no conceptual analysis can meet the requirements both of correctness and of informativeness. If Plato says "If you make a false statement, I will throw you in the water", and Socrates responds, "You will throw me in the water", there is no way for Plato to keep his promise. How can people experience strong emotions from purely fictional things? If all truths are knowable, then all truths must in fact be known. Paradox of free will: If God knows in advance how we will decide, how can there be free will? Why can induction be used to confirm that things are "green", but not to confirm that things are "grue"? When one pursues happiness itself, one is miserable; but, when one pursues something else, one achieves happiness. If asking oneself "Am I dreaming? A paradoxical game between two players, one of whom can predict the actions of the other. Several distinct paradoxes share this name. Can an omnipotent being create a rock too heavy for itself to lift? The author of a book may be justified in believing that all his statements in the book are correct, at the same time believing that at least one of them is incorrect. Epicurean paradox The existence of evil seems to be incompatible with the existence of an omnipotent, omniscient, and morally perfect God. Even though rules are intended to determine actions, "no course of action could be determined by a rule, because any course of action can be made out to accord with the rule". When a white horse is not a horse: White horses are not horses because white and horse refer to different things. In Kabbalah , how to reconcile self-awareness of finite Creation with Infinite Divine source, as an emanated causal chain would seemingly nullify existence. Economics paradoxes One class of paradoxes in economics are the paradoxes of competition , in which behavior that benefits a lone actor would leave everyone worse off if everyone did the same. These paradoxes are classified into circuit, classical and Marx paradoxes. A book arguing that antitrust enforcement artificially raised prices by protecting inefficient competitors from competition. To sell information you need to give it away before the sale. Two players reaching a state of Nash equilibrium both find themselves with no profits gained via exploitation. Adding extra capacity to a network can reduce overall performance. Consumption varies surprisingly smoothly despite sharp variations in income. Increasing road capacity at the expense of investments in public transport can make overall congestion on the road worse. For countries with income sufficient to meet basic needs, the reported level of happiness does not correlate with national income per person. With capacity constraints, there may not be an equilibrium. The perceived failure of European countries to translate scientific advances into marketable innovations. Why were interest rates and prices correlated? Increasing the price of bread makes poor people eat more of it. Inability to recoup cost of obtaining market information implies efficient markets cannot exist. Some businesses bring about their own downfall through their own successes. Increases in efficiency lead to even larger increases in demand. Some countries export labor-intensive commodities and import capital-intensive commodities, in contradiction with the Heckscherâ€”Ohlin theorem. Paradox of luxury goods. The more expensive some commodity is, less it is used after acquiring. Capital is not flowing from developed countries to developing countries despite the fact that developing countries have lower levels of capital per worker, and therefore higher returns to capital. Actions that may be vicious to individuals may benefit society as a whole. Keeping everyone out of an information system is impossible, but so is getting everybody in. The imposition of a tariff on imports may reduce the relative internal price of that good. Why do generations that significantly improve the economic climate seem to generally rear a successor generation that consumes rather than produces? If everyone saves more money during times of recession, then aggregate demand will fall and will in turn lower total savings in the population. If everyone tries to work during times of recession, lower wages will reduce prices, leading to more deflationary expectations, leading to further thrift, reducing demand and thereby reducing employment. Paradox of value , also known as diamond-water paradox: Water is more useful than diamonds, yet is a lot cheaper. Worker productivity may go down, despite technological improvements. Using the Kaldorâ€”Hicks criterion , an allocation A may be more efficient than allocation B, while at the same time B is more efficient than A. Successfully fixing a problem with a defective product may lead to higher consumer satisfaction than in the case where no problem occurred at all. People will only offer a modest fee for a reward of infinite expected value.

Chapter 4 : Liar Paradox | Internet Encyclopedia of Philosophy

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References and Further Reading 1. The Horned Man is a version of the "When did you stop beating your wife? This is not a simple question, and needs a carefully phrased reply, to avoid the inevitable come-back to "I have not. It is a question of what the "not," or negation means, in this case. If "stopped beating" means "beat before, but no longer," then "not stopped beating" covers both "did not beat before" and "continues to beat. However, your audience might still need to be taken slowly through the alternatives before they clearly see this. Likewise with the Horned Man, which arises if someone wants to say, for instance, "what you have not lost you still have. The Heap is nowadays commonly referred to as the Sorites Paradox, and concerns the possibility that the borderline between a predicate and its negation need not be finely drawn. We would all say that a man with no hairs on his head was bald, and that a man with, say, 10, hairs on his head was hirsute, that is not bald, but what about a man with only 1, hairs on his head, which are, say, evenly spread? It is not too clear what we should say, although maybe some would still want to say positively "bald," while others would want to say "not bald. The lazy solution says that any lack of certainty about what to say is merely a matter of us not having yet decided upon, or even having the need to make up our mind about, a "precisification" of the concept of baldness. There are objectors to this "epistemic" way of seeing the matter, some of whom would prefer to think, for instance see, e. Sainsbury , that there was something essentially "fuzzy" about baldness, so it is a "vague predicate" by the nature of things, instead of just through lack of effort, or need. For recent work in this area, see, for instance, Williamson , and Keefe The Hooded Man is about the concept of knowledge, and in other versions has again been much studied in recent years, as we shall see. In its original version the problem is this: For we can distinguish being acquainted with your brother from knowing that someone is your brother. Although you do not know it, you are certainly acquainted with the hooded man, since he is your brother, and you are acquainted with your brother. But that does not entail that you know that the hooded man is your brother, indeed, evidently you do not. We could also say, in that case, that you did not recognize your brother, for the notion of recognition is close to that of knowledge. Thus you might well be able to recognise your brother, but that does not require you can always do so, it merely means you can do better at this than those people who cannot do so. If we re-phrase the case: The basic idea had several variations, even in antiquity. Indeed a whole host of complications of The Liar have been constructed, especially in the last century, as we shall see. Now in The Cretan there is no real antinomy -- it may simply be false that all Cretans are liars; but if someone says just "I am lying," the situation is different. For if it is true that he is lying then seemingly what he says is false; but if it is false that he is lying then what he is saying may seem to be true. A pedant might say that "lying" was strictly not telling an untruth, but telling merely what one believes to be an untruth. The pedant, however, misses the point that his verbal nicety can be circumvented, and the paradox re-constructed in another, indeed many other forms. We shall look in more detail later at the paradox here, in some of its more complicated versions. For, Zeno argued see, for instance, Owen , and Salmon , if there were such units then they would either have a size, or not have a size. But if they had a size we would have the paradox of The Stadium, while if they had no size we would have the paradox of The Arrow. Thus if runners A and B are approaching one another both at unit speed, then, supposing the units have a finite size, after one time unit they will have each moved one space unit relative to the stadium. But they will have moved two space units relative to each other, which implies that there was a time unit in between when they were just one space unit apart. So the time unit must be divisible after all. On the other hand, if the units of division have no size, then, at any given time, an arrow in flight must occupy a space just equal to itself -- for it cannot move within that time. But if so then it is at rest, and the arrow never moves. That would seem to mean that space and time are divided without limit. But Zeno argued that if space and time were in themselves divided without limit then we would have the paradox of Achilles and the Tortoise. A runner, before he gets to the end of his race would have to get to the half-way point, but then also to the half-way point beyond that, that is the three-quarter-way point, and so on. There would be no limit to the sequence of points he would have to get to, and so there would always be a bit more to be run, and he could never get to the end. Likewise in a competitive race, even, say, between the super-speedy Achilles and a tortoise: Achilles would not be able to catch the tortoise up -- so long as the tortoise was given a start. Hence he never catches it up. No continuous magnitude, Aristotle thought, is actually composed of parts, since, although it may be divisible into parts without limit, the continuum is given before any such resulting division into parts. In particular, Aristotle denied that there could be any non-finite parts, and so is often called a "Finitist": This view came to be challenged later, since it means that an arrow can only be "at rest" if it is at the same place at two separate times -- for Aristotle both rest and motion can only be defined over a finite increment of time. But later the notion of an instantaneous velocity came to be accepted, and that includes the case where the velocity is zero. The puzzle about non-finite parts may remind one of the question which occupied many scholastic theologians in the Middle Ages: And it is perhaps no accident that the theorist who gave the currently received answer to the general question of how many things without any extension make up a whole which has such an extension was a fervent believer in God. Moving to Modern Times Between the classical times of Aristotle and the late nineteenth century when Cantor worked, there was a period in the middle ages when paradoxes of a logical kind were considered intensively. That was during the fourteenth century. Notable individuals were Paul of Venice, living towards the end of that century, and John Buridan, born just before it. As models of the care, and clarity which is required to extricate oneself from the above kind of difficulties with problem propositions each of these writers will surely stand forever. As an illustration, Buridan discusses "No change is instantaneous" in the following way Scott , p I prove it, because every change is either in an indivisible instant or it is in a divisible time. But none is in an indivisible instant, since an indivisible instant cannot be given in time, as is always supposed. Hence every change is in divisible time, and every such must be called temporal and not instantaneous. The opposite is argued, because at least the creation of our intellective soul is instantaneous. For since it is indivisible, it must be made altogether at once, not one part after another. And such creation we call instantaneous. I posit the case that you see your father coming from a distance, in such a way that you do not discern whether it is your father or another. Then it is proved, because you do indeed know your father, and he is the one approaching; hence, you know the one approaching. Likewise, you know him who is known by you, but the one approaching is known by you; hence, you know the one approaching. I prove the minor, because your father is known by you and your father is the one approaching; hence, the one approaching is known by you. The opposite is argued, because you do not know him of whom, if you are asked who he is, you will answer truly "I do not know. Broadly speaking, that is to say, Buridan made a distinction similar to that mentioned before, between general paradoxes of a logical nature, and "the logical paradoxes. And then Socrates utters only this proposition: Indeed, quite generally, sophisms about the nature of change and continuity, about knowledge and its objects, and the ones about the notion of self-reference, amongst many others, have attracted a great deal of very professional attention, once their significance was realised, with techniques of analysis drawn from developments in formal logic and linguistic studies being added to the careful and clear expression, and modes of argument found in the best writers before. The pace of change started to quicken in the later nineteenth century, but the one earlier thinker who will also be mentioned here is Bishop Berkeley, who was active in the early eighteenth. For a history of this period, in connection with the issues which concerned Berkeley, see, for instance, Grattan-Guinness It will be remembered that in the calculation of a derivative the following fraction is considered: The point was appreciated to some extent elsewhere. Leibniz, however, had no problem with the notion of an actual infinite division of a line -- or with the idea that the result could be a finite quantity. However, while Leibniz introduced finite infinitesimals instead of fluxions, this idea was also questioned as not sufficiently rigorous, and both ideas lost ground to definitions of derivatives in terms of limits, by Cauchy and Weierstrass in the nineteenth century. Leibniz would not have thought it too sensible to ask how many of his infinitesimals made up the line, but Cantor made much more precise the answer "infinitely many. We will look at those in the next section, which will then lead us into twentieth century developments in the area of self-reference. These issues, it will be remembered, centred on the problem of non-recognition, and in various ways two central cases of this have been given close attention since the end of the nineteenth century. A great deal of other relevant discussion has also gone on, but these two cases are perhaps the most important, historically see, for example, Linsky In fact the Morning Star is the same as the Evening Star, we now realise, but this was not always recognised, and indeed it is now realised that even the term "star" is a misnomer, both objects being the planet Venus. Still someone ignorant of the astronomical identity, it may be thought, might accept "The Evening Star is in the sky," but reject "The Morning Star is in the sky. Ortcutt, a respectable man with grey hair, once seen at the beach. In one location he was taken to be not a spy, in another place he was taken to be a spy, as one might say; but is that quite the best way the situation should be described? Maybe one who does not recognise him can have beliefs about the man at the beach without thereby having those beliefs about the respectable man with grey hair -- or even Bernard J. Certainly Quine thought so, which has not only caused a large scale controversy in itself; it has also led to, or been part of much broader discussions about identity in similar, but non-personal, intensional notions, like modality. Thus, as Quine pointed out, it would not seem to be necessary that the number of the planets is greater than 4, although it is necessary that 9 is greater than 4, and 9 is the number of the planets. A branch of formal logic, Intensional Logic, has been developed to enable a more precise analysis of these kinds of issue. Some Recent Logical Paradoxes It was developments in other parts of mathematics which were integral to the discovery of the next logical paradoxes to be considered. These were developments in the theory of real numbers, as was mentioned before, but also in Set Theory, and Arithmetic. Arithmetic is now taken to be concerned with a "denumerable" number of objects -- the natural numbers -- while real numbers are "non-denumerable. The tradition up to the middle of the nineteenth century did not look at these matters in this kind of way. For the natural numbers arise in connection with counting, for instance counting the cows in a field. If there are a number of cows in the field then there is a set of them: But with the beef in the field we do not normally talk in these terms: But continua from Cantor onwards have been seen as composed of non-finite individuals. And not only that is the change. For also the number of individuals in some set of individuals -- whether cows, or the non-finite elements in beef -- has been taken to be possibly non-finite, with a whole containing those individuals being then still available: We now commonly have the idea that there may be infinite sets first of finite entities, which will then be "countable" or "denumerable," but also there will be sets of non-finite, infinitesimal entities, which will be "uncountable," or "non-denumerable. Set Theory looked like it would become the entire foundation for mathematics. But if the result held for all predicates "F" then we could say, for any "F" there is a z such that:

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Chapter 5 : Examples of Paradox

This entry concentrates on the emergence of non-trivial logical themes and notions from the discussion on paradoxes from the beginning of the 20th century until , and attempts to assess their importance for the development of contemporary logic.

The original goes something like this: The Tortoise challenged Achilles to a race, claiming that he would win as long as Achilles gave him a small head start. Achilles laughed at this, for of course he was a mighty warrior and swift of foot, whereas the Tortoise was heavy and slow. Achilles laughed louder than ever. He knew he was the superior athlete, but he also knew the Tortoise had the sharper wits, and he had lost many a bewildering argument with him before this. Would you say that you could cover that 10 meters between us very quickly? And you would catch up that distance very quickly? Suppose I wish to cross the room. First, of course, I must cover half the distance. Then, I must cover half the remaining distance. Then I must cover half the remaining distanceâ€¦and so on forever. The consequence is that I can never get to the other side of the room. What this actually does is to make all motion impossible, for before I can cover half the distance I must cover half of half the distance, and before I can do that I must cover half of half of half of the distance, and so on, so that in reality I can never move any distance at all, because doing so involves moving an infinite number of small intermediate distances first. Now, since motion obviously is possible, the question arises, what is wrong with Zeno? What is the "flaw in the logic? Yet we know better. Rather than tackle Zeno head-on, let us pause to notice something remarkable. And before I can walk the remaining half-mile I must first cover half of it, that is, a quarter-mile, and then an eighth-mile, and then a sixteenth-mile, and then a thirty-secondth-mile, and so on. Well, suppose I could cover all these infinite number of small distances, how far should I have walked? An infinite sum such as the one above is known in mathematics as an infinite series , and when such a sum adds up to a finite number we say that the series is summable. Obviously, it will take me some fixed time to cross half the distance to the other side of the room, say 2 seconds. How long will it take to cross half the remaining distance? Half as longâ€"only 1 second. Covering half of the remaining distance an eighth of the total will take only half a second. And once I have covered all the infinitely many sub-distances and added up all the time it took to traverse them? Only 4 seconds, and here I am, on the other side of the room after all. And poor old Achilles would have won his race. Consider a lamp, with a switch. Hit the switch once, it turns it on. Hit it again, it turns it off. Let us imagine there is a being with supernatural powers who likes to play with this lamp as follows. First, he turns it on. At the end of one minute, he turns it off. At the end of half a minute, he turns it on again. At the end of a quarter of a minute, he turns it off. In one eighth of a minute, he turns it on again. And so on, hitting the switch each time after waiting exactly one-half the time he waited before hitting it the last time. Applying the above discussion, it is easy to see that all these infinitely many time intervals add up to exactly two minutes. At the end of two minutes, is the lamp on, or off? Here the lamp started out being off. Would it have made any difference if it had started out being on? Platonic Realms, 10 Apr Sidney 10 Apr

Chapter 6 : Paradoxes and Contemporary Logic (Stanford Encyclopedia of Philosophy)

A paradox is a statement that, despite apparently valid reasoning from true premises, leads to an apparently-self-contradictory or logically unacceptable conclusion. A paradox involves contradictory-yet-interrelated elements that exist simultaneously and persist over time.

Who shaves the barber? Discuss this paradox 4. If I am ill and it is my destiny to regain health, then I will regain health whether I visit a doctor or not. How could you question the presented opinion? My Favorite Sophisms 1. Crocodile Sophism A slim crocodile living in the Nile took a child. His mother begged to have him back. The crocodile could not only talk, but was also a great sophist and stated, "If you guess correctly what I will do with him, I will return him. Discuss this sophism 2. What is better - eternal bliss or a simple bread? What is better than eternal bliss? But a slice of bread is better than nothing. So a slice of bread is better than eternal bliss. The man who wrote such a stupid sentence can not write at all. Are you an analphabet? Write a letter and we will send you free of charge instructions how to undo it. What will happen if such a bullet hits such an armor? Can a man drown in the fountain of eternal life? Your mission is not to accept the mission. A girl goes into the past and kills her Grandmother. Since her Grandmother is dead, the girl was never born. If she were never born, she never killed her grandmother. If the temperature this morning is 0 degrees and the Weather Channel says, "it will be twice as cold tomorrow", what will the temperature be? Answer truthfully yes or no to the following question: What happens if you are in a car going the speed of light and you turn the headlights on? I conclude with this challenge: Let the God Almighty create a stone, which he is not capable of lifting!

Chapter 7 : Famous Paradoxes - Examples and Definition

What exists, should be seen! Or, should it not? Lets discover some of the things by which this awesome UNIverse is made up of!

DP] This version of the Liar is one of many. With a little more complexity, for instance, either capture or release can be avoided in favor of some other background assumptions. Intuitionistic variants of the Liar are also available, though we shall not explore intuitionistic logic here. We invoke EFQ to finish the proof. EFQ is the principle that every sentence follows from a contradiction; it sanctions the step from a single contradiction to outright triviality of logic. In the face of such absurdity triviality , we conclude that something is wrong in the foregoing Liar reasoning. This, in the end, is the question that the Liar paradox raises. Significance We have now seen that with some elementary assumptions about truth and logic, a logical disaster ensues. What is the wider significance of such a result? From time to time, the Liar has been argued to show us something far-reaching about philosophy. McGee and others suggest that the Liar shows the notion of truth to be a vague notion. Glanzberg holds that the Liar shows us something important about the nature of context dependence in language, while Eklund holds that it shows us something important about the nature of semantic competence and the languages we speak. Gupta and Belnap claim that it reveals important properties of the general notion of definition. And there are other lessons, and variations on such lessons, that have been drawn. Of more immediate concern, at least for our purposes here, is what the Liar shows us about the basic principles governing truth, and about logic. In a skeptical vein, Tarski himself , seems to have thought the Liar shows the ordinary notion of truth to be incoherent, and in need of replacement with a more scientifically respectable one. More common, and perhaps the dominant thread in the solutions to the Liar, is the idea that the basic principles governing truth are more subtle than the T-schema reflects. The Liar has also formed the core of arguments against classical logic, as it is some key features of classical logic that allow capture and release to result in absurdity. Notable among these are the arguments for logics that are paracomplete e. However, Ripley b argues that classical logic can be maintained while shedding the features in question. In many cases inspired by wider views of the significance of the paradox, there have been a number of attempts to one way or another resolve the paradox. It is to these proposed solutions that we now turn. Some Families of Solutions In this section, we briefly survey some approaches to resolving the Liar paradox. We group proposed solutions into families, and try to explain the basic ideas behind them. In many cases, a full exposition would involve a great deal of technical material, that we will not go into here. Interested readers are encouraged to follow the references we provide for each basic idea. The main idea is that the principles of capture and release are the fundamental conceptual principles governing truth, and cannot be modified. One important way to motivate non-classical solutions is to appeal to a form of deflationism about truth. Such views take something akin to the T-schema to be the defining characteristic of truth, and as such, not open to modification see, e. Holding capture and release fixed, and applying it to all sentences without restriction, yields triviality unless the logic is non-classical. There are two main sub-families of non-classical transparency truth theories: We sketch the main ideas of each. As a result, the logic of truth is non-classical. This idea is perhaps most natural in response to the simple-falsity Liar. But this will not suffice, for instance, for the simple-untruth Liar. This says nothing about falsity. There have been many proposals for using such non-classical logics to address the Liar. An early example is van Fraassen , Among many such logics are a number of three-valued logics that allow sentences to take a third value over and above true and false. Sentences like Liar sentences take the third value. For more details, see the entry on many-valued logic , or Priest First and foremost, we have: One way of understanding the important work of Kripke and related work of Martin and Woodruff is as a way of achieving just that. Recall, we are assuming a language comes equipped with a valuation scheme. Thus, it is a self-applicative truth predicate as the deflationist-inspired picture we mentioned must require , even though we begin with a language without a truth predicate. The main innovation is to see the truth predicate as partial. Rather than simply having an extension, it has an extension set of things of which it is true , and an anti-extension set of things of which it is false. See, for instance, Burgess and McGee for discussion. Kripke-style constructions engage a fair bit of mathematical subtlety. For an accessible overview of more of the details, see Soames For a more mathematically rich exposition, see McGee This reveals a limitation of the Kripkean approach to the Liar. The Kripke construction at hand, then, thus fails to enjoy all T-biconditionalsâ€"the natural candidates for expressing in the theory the basic capture and release features of truth. See Field , and further discussion in Beall This has recently played a key role in a number of discussions of conditionals and paradoxes; see for example Beall et al. We sketched a paracomplete option above. We now turn to a paraconsistent option. Here, the basic idea is to allow the contradiction e. But paraconsistent approaches have also found motivation in a Dummett-inspired anti-deflationist view, which takes the role of truth as the aim of assertion seriously cf. Hence, according to any such dialetheic line according to which at least one sentence is both true and not true , the only option is to reject EFQ. Dialetheism Priest , has been one of the leading voices in advocating a paraconsistent approach to solving the Liar paradox. This is what Priest calls the dialetheic approach to truth. See the entry on dialetheism for a more extensive discussion. Thus, sentences in LP can be both true and false. However, as we discuss further in section 4. This implements the idea of gluts, as the earlier version implemented the idea of gaps. This construction was not given by Kripke himself, but variants have been pursued by a number of authors, including Dowden , Leitgeb , Priest , , Visser , and Woodruff Combining paracompleteness and paraconsistency Though we have identified paracomplete and paraconsistent approaches to the Liar as two distinct options, they are not incompatible. For general discussion of such frameworks, see, e. More generally, he took the lesson of the Liar to be that languages cannot express the full range of semantic concepts that describe their own workings. One of the main goals of the non-classical approaches to the Liar we have surveyed here is to avoid this conclusion, which many have seen as far too drastic. However, how successful these approaches have been in this regard remains a highly contentious issue. In one sense, both the paracomplete and paraconsistent approaches achieve the desired result: In this respect, they both present languages which contain their own truth predicate. In the paracomplete case, the issue of whether this suffices has been much debated. One further point follows from this. Hence, as we mentioned, the status of gaps and gluts can be complicated. For the issue of revenge, the key problem is simply that the paracomplete approach cannot accurately state its own solution to the Liar. Just what to make of this has been debated. Because of this, some authors, such as McGee , T. Parsons , and Soames have in effect maintained that the Liar sentence failing to be true is a further fact that is goes beyond what the truth predicate needs to express, and so is immaterial to the success of the solution to the Liar. It thus has been argued to fail to achieve a fully adequate theory of truth. Kripke himself notes that there are some semantic concepts that cannot be expressed, and the argument has been pressed by C. One way of spelling out what is missing in the paracomplete language is to introduce a new notion of determinateness, so that the status of the Liar is that of not being determinately true. If so, then the Kripke paracomplete language cannot express this concept of determinateness. Some approaches taking paracomplete ideas on board have sought to supplement the Kripke approach by adding notions of determinate truth. McGee does so in a basically classical setting. See also some of the papers in Beall ed. LP theories can state this. On the other hand, some such as Littmann and Simmons and S. Shapiro , have thought that there is a dual problem: Some put this alleged problem as the problem of characterizing being just true. Whether this is a problem is something we leave open. For some discussion, see Field and Priest We can illustrate this with the simple-falsity Liar. Suppose one starts with this as the bench-mark Liar paradox, and proposes a simple solution that rejects bivalence. In response one is shown the simple-untruth Liar, which undercuts the simple solution. Revenge paradoxes for paracomplete solutions are often proposed: Failing to correctly state the status of the simple-untruth Liar is one example.

Chapter 8 : Logical Paradoxes | Internet Encyclopedia of Philosophy

A paradox is a statement that may seem absurd or contradictory but yet can be true, or at least makes sense. Paradoxes are often contrary to what is commonly believed and so play an important part in furthering our understanding in literature and everyday life, or they can simply be an entertaining brain teaser.

Q from 3 and 4 The consequence is outrageous. That is why the paradox is a serious problem. An appropriate reaction to any paradox is to look for some unacceptable assumption made in the apparently convincing argument or else to look for a faulty step in the reasoning. Only very reluctantly would one want to learn to live with the contradiction being true, or ignore the contradiction altogether. By the way, what this article calls "paradoxes" are called "antinomies" by Quine, Tarski, and some other authors. We naturally want our theory of truth not to allow paradoxes. Aristotle offered what most philosophers consider to be a correct, necessary condition for any adequate theory of truth. A sentence is true if, and only if, what it says is so. In his article, "The Concept of Truth in formalized Languages," Tarski rephrased the idea this way: A true sentence is one which says that the state of affairs is so and so, and the state of affairs indeed is so and so. Before we say more about the trouble with our theories of truth and reference, it will be helpful to describe the use-mention distinction. This is the distinction between using a term and mentioning it. Lassie is a helpful dog, but "Lassie" is a name, not a dog. Placing pairs of quotation marks around a term serves to name or mention it. The use-mention distinction applies to sentences as well as terms. Similarly, if the same sentence about snow were named or mentioned not with quotation marks but with the numeral 88 inside a pair of parentheses, then 88 would be true just in case snow is white. There is still another way to refer to sentences, namely via self-reference. If I say, "This sentence is written in English, and not Italian," then the phrase "This sentence" refers to that sentence. This is all straightforward, and is a well-accepted way of doing naming and referring. There is another important point to make about the use of quotation marks. It is a remark about sentences. If we have two names with the same denotation, then usually one name can be substituted for the other in a sentence without the newly-produced sentence changing its truth-value. The substitution preserves truth. There are well known exceptions to this substitution principle. For example, suppose this is true: John said, "Mark Twain was not a famous 21st century U. So, in substituting we need to be careful about substituting inside a quoted phrase. All these remarks about truth, reference, and substitution seem to be straightforward and not troublesome. Unfortunately, together they do lead to trouble, and the resolution of the difficulty is still an open problem in philosophical logic. Here is what Tarski is requiring. If we want to build a theory of truth for English, and we want to state the theory using English, then the theory must entail the specific T-sentence: If we want instead to build a theory of truth for German and use English to state the theory, then the theory should, among other things, at least entail the T-sentence: A great many philosophers believe Tarski is correct when he claims his Convention T is a necessary condition on any successful theory of truth for any language. However, do we want all the T-sentences to be entailed and thus come out true? Probably not the T-sentence for the Liar Sentence. Substituting the latter for L on the right of the above biconditional yields the contradiction: That is the argument of the Liar Paradox, very briefly. The proof requires the following additional assumptions. Here is a quotation from Tarski We have implicitly assumed that the language in which the antinomy is constructed contains, in addition to its expressions, also the names of these expressions, as well as semantic terms such as the term "true" referring to sentences of this language; we have also assumed that all sentences which determine the adequate usage of this term can be asserted in the language. A language with these properties will be called "semantically closed. We have assumed that in this language the ordinary laws of logic hold. But if so, then one can eventually deduce a contradiction. This deduction by Tarski is a formal analog of the informal argument of the Liar Paradox. The contradictory result tells us that the argument began with a false assumption. According to Tarski, the error that causes the contradiction is the assumption that the global truth predicate can be well-defined. In , he created the first formal semantics for quantified predicate logic. Here are two imperfect examples of how he partly defines truth. For example, we might formalize the English sentence, "Alfred is fat", by translating it as Fa; then Tarski is telling us that Alfred is fat just in case Alfred is a member of the set of all things that are fat. These two definitions are still imprecise because the appeal to the concept of property should be eliminated, and the definitions should appeal to the notion of formulas being satisfied by sequences of objects. Tarski then took on the project of discovering how close he could come to having a well-defined truth predicate within a classical formal language without actually having one. That project, his hierarchy of meta-languages, is also his key idea for solving the Liar Paradox. Overview of Ways Out of the Paradox a. Five Ways Out There are many proposed solutions to the paradox. All other things being equal, adopting simple, intuitive and conservative semantic principles is to be preferred ideally to adopting ad hoc, complicated and less intuitive semantic principles that have many negative consequences. The same goes for revision of a concept or revision of a logic. Nor should we try to find a way by declaring that we must adhere to the principle, "Avoid paradoxes. That is also an implausible route because Alfred Tarski showed that by using a vagueness-free formal language he could produce the Liar Paradox. Maybe the route to a solution is to uncover some subtle equivocation in our concepts employed in producing the contradiction. There have been many suggestions along this line, but none have been widely accepted. Or perhaps we should simply accept that there is a contradiction unless we make appropriate changes. Because the Liar Paradox depends crucially upon our ideas about how to make inferences and how to understand the key semantic concepts of truth, reference, and negation, one might reasonably suppose that one of these needs revision. But we should proceed cautiously. No one wants to solve the Paradox by being heavy-handed and jettisoning more than necessary. Also, we should beware that any changes we do make might have their own drawbacks. If we adopt the metaphor of a paradox as being an argument which starts from the home of seemingly true assumptions and which travels down the garden path of seemingly valid steps into the den of a contradiction, then a solution to the Liar Paradox has to find something wrong with the home, find something wrong with the garden path, or find a way to live within the den. Less metaphorically, the main ways out of the Paradox are the following: The Liar Sentence is ungrammatical and so has no truth value yet the argument of the Liar Paradox depends on it having a truth value. The Liar Sentence is grammatical but meaningless and so has no truth value. The argument of the Liar Paradox is acceptable, and we need to learn how to live with the Liar Sentence being both true and false. Two philosophers might take the same way out, but for different reasons. In presenting any of these five proposed solutions to the Paradox, it is helpful to explore the details and the implications. There are many suggestions for how to deal with the Liar Paradox, but most are never developed to the point of giving a detailed theory that can speak of its own syntax and semantics with precision. The point of the philosophical justification is an unveiling of some hitherto unnoticed or unaccepted rule of language for all sentences of some category which has been violated by the argument of the Paradox. Perhaps the best we can do is to have a variety of ways out, some of which are better than some others in certain respects. That point should be kept in mind when this article cavalierly speaks of "the" way out. Symptomatic relief is sufficient. He says it may appear legitimate, at first, to admit that the Liar Sentence is meaningful and also that it is true or false, but the Liar Paradox shows that one should retract this admission and either just not use the Liar Sentence in any arguments, or say it is not really a sentence, or at least say it is not one that is either true or false. Wittgenstein is not particularly concerned with which choice is made. And, whichever choice is made, he believes it need not be backed up by any theory that shows how to systematically incorporate the choice. He treats the whole situation cavalierly and unsystematically. Most logicians want systematic removal of the Paradox. Disagreeing with Wittgenstein, P. He says such an investigation will reveal that the Liar Sentence is meaningful but fails to express a proposition. We are not ascribing a property to the proposition such as the property of correspondence to the facts, or coherence, or usefulness. We are performing an action of that sort. The person who utters the Liar Sentence is making a pointless utterance. Some proponents of their own favorite solution to the paradox agree that a systematic approach to the paradox is valuable, and they point out that in some formalism, say first-order arithmetic, the Liar argument cannot be reconstructed. For one example, perhaps the proponents will argue that the sub-argument from the Liar sentence being true to its being false is acceptable, but the sub-argument from the Liar sentence being false to its being true cannot be reconstructed in the formalism. From this they conclude that the Liar sentence is simply false and paradox-free. This may be the key to solving the Paradox, but it is not successful if there is no satisfactory response to the complaint that perhaps their reconstruction using that formalism shows more about the inadequacy of the formalism than the proper way out of the paradox. Sentences, Statements, and Propositions The Liar Paradox can be expressed in terms of sentences, statements, and propositions. The Strengthened Liar might begin with This sentence is not true. This statement is not true. This proposition is not true. This is not true. What are the important differences? Sentences are linguistic expressions, whereas statements and propositions are not. We sometimes use sentences to make statements and assert propositions, but we sometimes use sentences to ask questions and to threaten our enemies.

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Chapter 9 : Paradox - Wikipedia

Logical Paradoxes A paradox is generally a puzzling conclusion we seem to be driven towards by our reasoning, but which is highly counterintuitive, nevertheless. There are, among these, a large variety of paradoxes of a logical nature which have teased even professional logicians, in some cases for several millennia.

Introduction Between the end of the 19th century and the beginning of the 20th century, the foundations of logic and mathematics were affected by the discovery of a number of difficultiesâ€"the so-called paradoxesâ€"involving fundamental notions and basic methods of definition and inference, which were usually accepted as unproblematic. Since then paradoxes have acquired a new role in contemporary logic: Several basic notions of logic, as it is presently taught, have reached their present shape at the end of a process which has been often triggered by various attempts to solve paradoxes. This is especially true for the notions of set and collection in general, for the basic syntactical and semantical concepts of standard classical logic logical languages of a given order, the notion of satisfiability, definability. Early work on paradoxes of particular importance pertained to the following notions: Difficulties involving ordinal and cardinal numbers The earliest modern paradoxes concerned the notions of ordinal and cardinal number. The father of set theory, Cantor, had noticed similar difficulties already in as witnessed by Bernstein and by letters to Hilbert and Dedekind. Roughly, the former is a collection that cannot be an element of other collections, whereas the latter is a small collection, which can be an element of other collections see also the entry on the early development of set theory. In the case of the difficulty discovered by Burali-Forti, the consequence for Cantor was that the multiplicity Mannigfaltigkeit of ordinal numbers is itself well-ordered, but is not a set: It is strikingly simple, involves only predicate application, and it has an explicit self-referential reflexive character. From each answer, the opposite follows. Likewise, there is no class as a totality of those classes which, each taken as a totality, do not belong to themselves. From this I conclude that under certain circumstances a definable collection does not form a totality. However, there are objects that are not ranges of significance; these are just atoms i. The next type consists of classes or ranges of individuals; then one has classes of classes of objects of the lowest type, and so on see also the entry on type theory.. New difficulties still arise if one accepts that propositions form a type as they are the only objects of which it can be meaningfully asserted that they are true or false. Then we can inject types of propositions into propositions by means of the notion of logical product. Of course, if one were to adopt the extensional point of view, and hence identify equivalent propositions, the contradiction above could not be derived. Russell, however, sticks to an intensional point of view, stressing that equivalent propositions often can be quite different. So one is apparently forced to reject the assumption that propositions form one type, and hence to require that they ought to have various types, while logical products ought to have propositions of only one type as factors. This was eventually the basis of the ramified theory of types, but in Russell still regarded the suggestion as harsh and artificial. As the footnote on p. In addition, Hilbert had noticed in unpublished work see Kahle and Peckhaus that additional contradictions of a mathematical nature can arise. The argument is based on functional self-application and hence direct self-reference. The contradiction is obtained by assuming that the universe includes everything, i. Then one introduces a new operation universal application in our sense: The results of the Hilbert school were not published because contradictions and paradoxes were regarded as symptoms of growth and as temporary difficulties. The diagnosis was that traditional logic is insufficient and the theory of concept-formation needs to be sharpened. Indeed, the new contradictions not only affected the conception of set and logical concepts, they also came to include the notion of definability and its relation with a fundamental issue: Consider the reals which are definable in finitely many words. They form a countable sequence: Since the continuum is uncountable, there exist reals not occurring in the given enumeration. So it appears that the definition is viciously circular, and that makes the definition illusory. According to him, this is not a restriction sinceâ€"he claimsâ€"there are nondenumerably many computable reals in his sense. He also states that it is possible to display the new computable continuum in a hierarchy i. This question was ultimately connected with the problem of understanding the atomistic continuum. The arithmetized conceptionâ€"in the sense of Dedekind or Cantor, where real numbers are identified with suitable sets of rationalsâ€"shifted the attention towards arbitrary infinite sequences of natural numbers. But this notion was not so easy to accept. According to Bernstein Bernstein a, p. But what is a rule? Since one must be liberal in order not to have just special classes of reals, if one is too restrictive , one is naturally driven to think of arbitrary finitely described laws, shifting the attention towards the syntax of the rules. The need for a specification of infinite sets is crucial in the discussion of the related well-ordering problem. It also affects the related issue of the classification of discontinuous functions and analytically representable functions of real variables, tackled by the French semi-intuitionists Borel, Baire and Lebesgue. Paradoxes, predicativism and the doctrine of types: Even more important for the history of mathematical logic, fundamental technical advances for solving the paradoxes and shaping the foundations of logic and mathematics were made: According to him, number-theoretic induction and the axiom of choice constitute independent intuitions, truly synthetic a priori judgements. His objection is that the definition is not admissible since it refers essentially to a totality to which the class to be defined belongsâ€"the definition is impredicativeâ€"and hence it is to be regarded as circular. In the later period, he advanced a novel approach to predicativity, which, though informally sketched, is suggestive of later developments in definability and proof theory see Feferman , Heinzmann He no longer insisted on the vicious circularity of the definition involved in the contradictions; instead, he held the view that a predicative classification is characterized by invariance, i. The paradoxes prove that a propositional function may be well-defined for every argument, and yet the collection of the values for which it is defined need not be a class. So the crucial problem becomes logical: Whatever comprises an apparent variable should not be one among the possible values of that variable. Of course, the vicious circle principle is not itself a theory, but a condition any adequate theory has to satisfy. Russell , tentatively proposed three alternative approaches: In the no-classes theory, classes are not independent entities and anything said about them is to be regarded as an abbreviation of a statement about their members and the propositional functions defining them. As far as we know, it is exactly at this point in time that the probably most cited semantical puzzle in the history of logic regains a conspicuous position in logical analysis. In his analysis of the Liar paradox, Russell assumed that there exists a true entityâ€"the propositionâ€"that is presupposed by a genuine statement e. The same holds if the statement is false, but not in the case where the statement itself contains quantified variables. So the conclusion is that the Liar is false because it does not state a proposition. Similar considerations apply to the paradox suggested by Berry, which is briefly stated in Russell for the first time in published form, and has the merit of not going beyond the domain of finite numbers. Consider the natural numbers that are definable by means of less than 18 syllables: So there exist numbers which are not definable using less than 18 syllables. Consider the least such number: It was first developed by Russell in the fundamental memoir Mathematical Logic as based on the theory of types of The doctrine of types is based upon the observation that universal quantificationâ€"understood as full generality, i. The essential point is that each propositional function has a range of significance, i. Formally speaking, each variable must have a preassigned type. The paradoxes or reflexive fallacies prove that certain collections, such as the totality of all propositions, of all classes and so on, cannot be types. Therefore, the logical entities divide into types, and, in particular, every propositional function must have a higher type than its arguments. Moreover, in the light of the vicious circle principle, the notion of order must also be introduced. No object can be defined by quantifying over a totality which contains the object itself as element; hence the order of every propositional function must be greater than the order of propositional functions over which it quantifies. The main idea is clarified in Russell pp. First of all, there are elementary propositions, i. Individuals are entities without logical structure and can be regarded as the subjects of elementary propositions. The second logical type embraces the first order propositions, i. Quantification over first order propositions gives rise to a new type, consisting exactly of second order propositions. So, for instance, a function which applies to individuals and takes first-order propositions as values is first-order. Then the Liar is simply false rather than contradictory; and this solves the paradox. Similar arguments solve the other paradoxes. In this theory, predicative functions of one argument, i. For instance, a predicative function of an individual variable must have order 1 in current terminology, it is elementarily definable and quantifies only over individuals. The axiom of reducibility AR states that every propositional function is equivalent, for all its values, to a predicative function of the same variables. Thus, according to the axiom of reducibility, statements about arbitrary functions can be replaced by statements about predicative functions; and predicative functions play the role of classes, i. Besides the axiom of infinity, AR is an essential tool for reconstructing classical mathematics, but it is a strong existential principle, apparently in conflict with the philosophical idea that logical and mathematical entities are to be constructively generated according to the vicious circle principle. Nonetheless, it was adopted in the first edition of the monumental Principia Mathematica, written in collaboration with A. Whitehead and published in vol. Other important applications of ramified hierarchies have been given since the late fifties in different fields from recursion theory to proof theory; see the entry on type theory. Some of the more stimulating and original proposals are surveyed in the rest of this section. For instance, there is no precise criterion for deciding whether a given expression of the natural language represents a rule uniquely defining a number. In spite of that, Peano elaborated a formal solution. According to him, sets are generated and, once formed, are conceptually invariant. When a new set is built up, it is added to those that have been used in forming it, without altering their pre-existent structure. His view can be seen as hinting at a sort of iterative conception of sets. It can be found in the dissertation of , chapter III cf. For instance, in order to decide whether a class falls under a propositional function, the class has to be a completed totality. The contradictions show that there are propositional functions which define complementary disjoint classes and yet do not satisfy the tertium non datur. Mathematically speaking, the contradiction can be avoided by denying that the largest well-ordered type has a successor order type p. Among the French mathematicians, the semi-intuitionist Borel introduced the distinction between effectively enumerable sets and denumerable ones. Borel has in mind a particular example: