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Chapter 1 : Optimal Control of Hybrid Systems | Max Planck Institute for Dynamics of Complex Technical S

Optimal control theory deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved.. It is an extension of the calculus of variations, and is a mathematical optimization method for deriving control policies.

Some important contributors to the early theory of optimal control and calculus of variations include Johann Bernoulli , Isaac Newton , Leonhard Euler , Ludovico Lagrange , Andrien Legendre , Carl Jacobi , William Hamilton , Karl Weierstrass , Adolph Mayer , and Oskar Bolza Some important milestones in the development of optimal control in the 20th century include the formulation dynamic programming by Richard Bellman in the s, the development of the minimum principle by Lev Pontryagin and co-workers also in the s, and the formulation of the linear quadratic regulator and the Kalman filter by Rudolf Kalman b. See the review papers Sussmann and Willems and Bryson for further historical details. Optimal control and its ramifications have found applications in many different fields, including aerospace, process control, robotics, bioengineering, economics, finance, and management science, and it continues to be an active research area within control theory. Before the arrival of the digital computer in the s, only fairly simple optimal control problems could be solved. The arrival of the digital computer has enabled the application of optimal control theory and methods to many complex problems. Formulation of optimal control problems There are various types of optimal control problems, depending on the performance index, the type of time domain continuous, discrete , the presence of different types of constraints, and what variables are free to be chosen. The formulation of an optimal control problem requires the following: Continuous time optimal control using the variational approach General case with fixed final time and no terminal or path constraints If there are no path constraints on the states or the control variables, and if the initial and final times are fixed, a fairly general continuous time optimal control problem can be defined as follows: Note that equation 2 represents the dynamics of the system and its initial state condition. Problem 1 as defined above is known as the Bolza problem. Calculus of variations Gelfand and Fomin, is concerned with the optimisation of functionals, and it is the tool that is used in this section to derive necessary optimality conditions for the minimisation of J u. Equation 5 is known as the co-state or adjoint equation. Equation 6 and the initial state condition represent the boundary or transversality conditions. These necessary optimality conditions, which define a two point boundary value problem , are very useful as they allow to find analytical solutions to special types of optimal control problems, and to define numerical algorithms to search for solutions in general cases. Moreover, they are useful to check the extremality of solutions found by computational methods. Sufficient conditions for general nonlinear problems have also been established. Distinctions are made between sufficient conditions for weak local, strong local, and strong global minima. Sufficient conditions are useful to check if an extremal solution satisfying the necessary optimality conditions actually yields a minimum, and the type of minimum that is achieved. See Gelfand and Fomin, , Wan, and Leitmann, for further details. The theory presented above does not deal with the existence of an optimal control that minimises the performance index J. See the book by Cesari which covers theoretical issues on the existence of optimal controls. The linear quadratic regulator A special case of optimal control problem which is of particular importance arises when the objective function is a quadratic function of x and u, and the dynamic equations are linear. The resulting feedback law in this case is known as the linear quadratic regulator LQR. The performance index is given by: In this case, using the optimality conditions given above, it is possible to find that the optimal control law can be expressed as a linear state feedback: This is an important result, as the linear quadratic regulator provides a way of stabilizing any linear system that is stabilizable. It is worth pointing out that there are well established methods and software for solving the algebraic Ricatti equation This facilitates the design of linear quadratic regulators. A useful extension of the linear quadratic regulator ideas involves modifying the performance index 8 to allow for a reference signal that the output of the system should track. Moreover, an extension of the LQR concept to systems with gaussian additive noise, which is known as the linear quadratic gaussian LQG controller, has been widely applied. The LQG controller involves coupling the linear quadratic regulator with the Kalman filter using the separation principle. See Lewis and Syrmos, for further details. Case with terminal constraints In case problem 1 is also subject to a set of terminal constraints of the form: According to this principle, the Hamiltonian must be minimised over all admissible u for optimal values of the state and costate variables. Minimum time problems One special class of optimal control problem involves finding the optimal input u t to reach a terminal constraint in minimum time. This kind of problem is defined as follows. These are called singular arcs. Additional tests are required to verify if a singular arc is optimizing. A particular case of practical relevance occurs when the Hamiltonian function is linear in at least one of the control variables. In such cases, the control is not determined in terms of the state and co-state by the stationarity condition 7. See Bryson and Ho, and Sethi and Thompson, for further details on the handling of singular arcs. Computational optimal control The solutions to many optimal control problems cannot be found by analytical means. Over the years, many numerical procedures have been developed to solve general optimal control problems. With direct methods, optimal control problems are discretised and converted into nonlinear programming problems of the form: The decision vector y contains the control and state variables at the grid points. Other direct methods involve a decision vector y which contains only the control variables at the grid points, with the differential equations solved by integration and their gradients found by integrating the co-state equations, or by finite differences. Other direct methods involve the approximation of the control and states using basis functions, such as splines or Lagrange polynomials. There are well established numerical techniques for solving nonlinear programming problems with constraints, such as sequential quadratic programming Bazaraa et al, Direct methods using nonlinear programming are known to deal in an efficient manner with problems involving path constraints. See Betts for more details on computational optimal control using nonlinear programming. See also Becerra, for a straightforward way of combining a dynamic simulation tool with nonlinear programming code to solve optimal control problems with constraints. Indirect methods involve iterating on the necessary optimality conditions to seek their satisfaction. This usually involves attempting to solve nonlinear two-point boundary value problems, through the forward integration of the plant equations and the backward integration of the co-state equations. Examples of indirect methods include the gradient method and the multiple shooting method, both of which are described in detail in the book by Bryson Dynamic programming Dynamic programming is an alternative to the variational approach to optimal control. It was proposed by Bellman in the s, and is an extension of Hamilton-Jacobi theory. This principle serves to limit the number of potentially optimal control strategies that must be investigated. It also shows that the optimal strategy must be determined by working backward from the final time. Consider Problem 1 with the addition of a terminal state constraint In some cases, the HJB equation can be used to find analytical solutions to optimal control problems. Dynamic programming includes formulations for discrete time systems as well as combinatorial systems, which are discrete systems with quantized states and controls. See the books Lewis and Syrmos, , Kirk, , and Bryson and Ho, for further details on dynamic programming. Discrete-time optimal control Most of the problems defined above have discrete-time counterparts. These formulations are useful when the dynamics are discrete for example, a multistage system , or when dealing with computer controlled systems. In discrete-time, the dynamics can be expressed as a difference equation: Examples Minimum energy control of a double integrator with terminal constraint Consider the following optimal control problem. Replacing 21 into the state equation 18 , and integrating both states gives: Therefore, the optimal control is given by: B maximum altitude climbing turn manoeuvre This example is solved using a gradient method in Bryson, Here, a path constraint is considered and the solution is sought by using a direct method and nonlinear programming. Such a flight path may be of interest to reduce engine noise over populated areas located ahead of an airport runway. This manoeuvre can be formulated as an optimal control problem, as follows. The distance and time units in the above equations are normalised. To obtain meters and seconds, the corresponding variables need to be multiplied by There are two controls: Jr and Ho Y. Problems With Ordinary Differential Equations. Sethi S and Thompson G. Applications to Management Science and Economics. J and Willems J. Internal references Ian Gladwell Boundary value problem. Scholarpedia , 3 1: James Meiss Dynamical systems. James Meiss Hamiltonian systems. Howard Eichenbaum Memory. John Butcher Runge-Kutta methods. Philip Holmes and Eric T. Further reading Athans M. An Introduction to the

Theory and Its Applications. Journal of Computational and Applied Mathematics.

Chapter 2 : ACADO Toolkit: Optimal Control of Discrete-Time Systems

Abstract. For a class of discrete systems with delays and additional state-dependent constraints, a maximum principle is derived. The existing general results in the field of discrete optimal control theory make it possible to deal with this class of optimization problems in a straight-forward way.

Vadim Azhmyakov CINVESTAV Optimal Control of Hybrid Systems Hybrid optimal control problems are highly nontrivial, as one has to deal not only with the infinite dimensional optimisation problems related to the continuous dynamics, but also with a potential combinatorial explosion related to the discrete part. Because of the large number of potential applications, there has been considerable interest in optimal hybrid control problems. We have focused on some specific, but practically important, classes of hybrid systems and derived necessary conditions of optimality and efficient conceptual algorithms to solve the related problems. Attia, our group became interested in the theory of optimal hybrid control. Hybrid optimal control problems are highly nontrivial, as one has to deal not only with the infinite dimensional optimization problems related to the continuous dynamics, but also with a potential combinatorial explosion related to the discrete part. Because of the large number of potential applications, there has been considerable interest in optimal hybrid control problems, with important contributions from, e. One of the most convenient ways to deal with the problem is to formulate it as a sequential problem, i. In each interval, the discrete state remains constant, and the continuous dynamics is characterised by a set of ODEs. The former is often referred to as autonomous switching, the latter as controlled switching. We have focused on some specific, but practically important, classes of hybrid systems and derived necessary conditions of optimality and efficient conceptual algorithms to solve the related problems: In contrast to the work by Caines and coworkers, we derived necessary optimality conditions without recourse to the technique of needle variations. Instead we apply a generalized Lagrange multiplier rule [1]. This allows us to obtain necessary conditions for a weak minimum as opposed to the Maximum Principle, which gives necessary conditions for a strong minimum. The difference between the two types of minima is the norm used to compare two feasible trajectories. The weak necessary conditions of optimality are said to hold if the continuous trajectories associated with the same discrete state are compared in the sense of the infinity norm in contrast to a strong minimum, where the 1-norm is usually employed. The problem is first formulated as an abstract optimisation problem in an appropriate Sobolev space. The differential equations are considered as operators acting on Sobolev spaces, and the switching surfaces are embedded into the operator as equality constraints. A generalised Lagrange multiplier is then applied to extract the necessary conditions of optimality. As a second class, we have investigated hybrid systems with autonomous switching where discrete transitions are accompanied by instantaneous changes jumps in the continuous states and where these state jumps i. Hybrid systems with jumps in the continuous states are often referred to as impulsive hybrid systems. In a first step, necessary conditions of optimality are established based on a variational approach. For this, a smooth variation preserving the switching sequence for the discrete state is introduced around the optimal trajectory. Applying the Lagrange principle gives a sequence of boundary-value problems that need to be solved and an equality condition on the gradient of the cost functional with respect to the jump parameters. Closed form expressions of the gradient are then obtained using a parameter variation where the effects of parametric variation are propagated on the whole trajectory. An algorithm based on gradient descent techniques is then proposed together with some convergence results. The algorithm uses forward-backward integration of the system dynamics and the adjoint equations together with a pointwise update of the jump parameters. Details are provided in [2] [3]. As a third class, we have considered hybrid systems with autonomous switching, continuous control inputs and controlled state jumps. Using a simple transformation, the problem under study can be formulated as a hybrid systems with autonomous switching where jump parameters are considered as a part of the control. Based on the results in [1] , we develop a new set of necessary conditions of optimality [4]. A combination of the algorithm developed for the class of impulsive autonomous hybrid systems [2] [3] together with a gradient based approach [5] for updating the control can be used to extract both the continuous control signals and the controlled jump parameters. Finally, [6] discusses how a class of optimal control problems for switched systems that are affine in the control input can be treated as convex problems.

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Optimal equivalence between the bilinear system and the switched system is analyzed, which shows that any optimal control law can be equivalently expressed as a switching law. This specific switching law is most unstable for the switched system, and thus can be used to determine stability under arbitrary switching.

Chapter 4 : Optimal control - Scholarpedia

control law without gridding the state space. The solution to optimal control problems for discrete-time hybrid systems was ï¬•rst outlined by Sontag in [48]. In his plenary presentation [39] at the European Control Conference, Mayne.

Chapter 5 : Optimal control - Wikipedia

CONTROL OF DISCRETE-TIME STOCHASTIC SYSTEMS bility of this method of expressing the index of performance is discussed in detail in [1] and [3]. The first step in determining an optimal control policy is to designate a set of control policies which are admissible in a particular application.

Chapter 6 : Optimal Control - Frank L. Lewis, Vassilis L. Syrmos, Vassilis L.. Syrmos - Google Books

In this work, we consider the optimal control problem of linear quadratic regulation for discrete time-variant systems with single input and multiple input delays. An innovative and simple method to derive the optimal controller is given. The studied problem is first equivalently converted into a.

Chapter 7 : ECE Optimal Control

This tutorial explains how to setup a optimal control problems for discrete time systems. Mathematical Formulation of Discrete Time Systems. A discrete time system consists typically of a state sequence (xk) and an associated time sequence (tk) satisfying an iteration of the form.

Chapter 8 : Discrete-Time Inverse Optimal Control for Nonlinear Systems - CRC Press Book

Discrete systems with delays, necessary optimality condi- tions, discrete maximum principle, state and control constraints, state- dependent control region.