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Chapter 1 : Theory (mathematical logic) - Wikipedia

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Motivations There have been many attempts to define truth in terms of correspondence , coherence or other notions. However, it is far from clear that truth is a definable notion. In these cases definitional approaches to truth have to fail. By contrast, the axiomatic approach does not presuppose that truth can be defined. Instead, a formal language is expanded by a new primitive predicate for truth or satisfaction, and axioms for that predicate are then laid down. This approach by itself does not preclude the possibility that the truth predicate is definable, although in many cases it can be shown that the truth predicate is not definable. In semantic theories of truth e. This definition is carried out in a metalanguage or metatheory, which is typically taken to include set theory or at least another strong theory or expressively rich interpreted language. So semantic approaches usually necessitate the use of a metalanguage that is more powerful than the object-language for which it provides a semantics. As with other formal deductive systems, axiomatic theories of truth can be presented within very weak logical frameworks. These frameworks require very few resources, and in particular, avoid the need for a strong metalanguage and metatheory. Formal work on axiomatic theories of truth has helped to shed some light on semantic theories of truth. For instance, it has yielded information on what is required of a metalanguage that is sufficient for defining a truth predicate. Semantic theories of truth, in turn, provide one with the theoretical tools needed for investigating models of axiomatic theories of truth and with motivations for certain axiomatic theories. Thus axiomatic and semantic approaches to truth are intertwined. This entry outlines the most popular axiomatic theories of truth and mentions some of the formal results that have been obtained concerning them. We give only hints as to their philosophical applications. Quantification over definable properties can then be mimicked in a language with a truth predicate by quantifying over formulas. The reduction of properties to truth works also to some extent for sets of individuals. There are also reductions in the other direction: Tarski has shown that certain second-order existence assumptions e. The mathematical analysis of axiomatic theories of truth and second-order systems has exhibited many equivalences between these second-order existence assumptions and truth-theoretic assumptions. In particular, proof-theoretic equivalences described in Section 3. The equivalence between second-order theories and truth theories also has bearing on traditional metaphysical topics. The reductions of second-order theories i. But PA can be strengthened by adding this consistency statement or by stronger axioms. In particular, axioms partially expressing the soundness of PA can be added. These are known as reflection principles. The process of adding reflection principles can be iterated: The reflection principles expressâ€”at least partiallyâ€”the soundness of the system. The Global Reflection Principle for a formal system S states that all sentences provable in S are true: The truth predicate has to satisfy certain principles; otherwise the global reflection principle would be vacuous. Thus not only the global reflection principle has to be added, but also axioms for truth. If a natural theory of truth like T PA below is added, however, it is no longer necessary to postulate the global reflection principle explicitly, as theories like T PA prove already the global reflection principle for PA. One may therefore view truth theories as reflection principles as they prove soundness statements and add the resources to express these statements. Thus instead of iterating reflection principles that are formulated entirely in the language of arithmetic, one can add by iteration new truth predicates and correspondingly new axioms for the new truth predicates. Thereby one might hope to make explicit all the assumptions that are implicit in the acceptance of a theory like PA. The resulting theory is called the reflective closure of the initial theory. Feferman has proposed the use of a single truth predicate and a single theory KF , rather than a hierarchy of predicates and theories, in order to explicate the reflective closure of PA and other theories. KF is discussed further in Section 4. The relation of truth theories and iterated reflection principles also became prominent in the discussion of truth-theoretic deflationism see Tennant and the follow-up discussion. More refined axioms have also been discussed by deflationists. If truth can be explicitly defined, it can be eliminated, whereas an axiomatized notion of truth may and often does come with commitments that go beyond that of the base theory. If truth does not have any explanatory force, as some deflationists claim, the axioms for truth should not allow us to prove any new theorems that do not involve the truth predicate. Accordingly, Horsten , Shapiro and Ketland have suggested that a deflationary axiomatization of truth should be at least conservative. Thus a non-conservative theory of truth adds new non-semantic content to a theory and has genuine explanatory power, contrary to many deflationist views. Certain natural theories of truth, however, fail to be conservative see Section 3. According to many deflationists, truth serves merely the purpose of expressing infinite conjunctions. It is plain that not all infinite conjunctions can be expressed because there are uncountably many non-equivalent infinite conjunctions over a countable language. Since the language with an added truth predicate has only countably many formulas, not every infinite conjunction can be expressed by a different finite formula. The formal work on axiomatic theories of truth has helped to specify exactly which infinite conjunctions can be expressed with a truth predicate. Feferman provides a proof-theoretic analysis of a fairly strong system. Again, this will be explained in the discussion about KF in Section 4. The base theory 2. There is an extensive philosophical discussion on the category of objects to which truth applies: Since the structure of sentences considered as types is relatively clear, sentence types have often been used as the objects that can be true. In many cases there is no need to make very specific metaphysical commitments, because only certain modest assumptions on the structure of these objects are required, independently from whether they are finally taken to be syntactic objects, propositions or still something else. The theory that describes the properties of the objects to which truth can be attributed is called the base theory. The formulation of the base theory does not involve the truth predicate or any specific truth-theoretic assumptions. The base theory could describe the structure of sentences, propositions and the like, so that notions like the negation of such an object can then be used in the formulation of the truth-theoretic axioms. Peano arithmetic has proved to be a versatile theory of objects to which truth is applied, mainly because adding truth-theoretic axioms to Peano arithmetic yields interesting systems and because Peano arithmetic is equivalent to many straightforward theories of syntax and even theories of propositions. However, other base theories have been considered as well, including formal syntax theories and set theories. Of course, we can also investigate theories which result by adding the truth-theoretic axioms to much stronger theories like set theory. Usually there is no chance of proving the consistency of set theory plus further truth-theoretic axioms because the consistency of set theory itself cannot be established without assumptions transcending set theory. In many cases not even relative consistency proofs are feasible. However, if adding certain truth-theoretic axioms to PA yields a consistent theory, it seems at least plausible that adding analogous axioms to set theory will not lead to an inconsistency. Therefore, the hope is that research on theories of truth over PA will give an some indication of what will happen when we extend stronger theories with axioms for the truth predicate. However, Fujimoto has shown that some axiomatic truth theories over set theory differ from their counterparts over Peano arithmetic in some aspects. The syntactical operations of forming a conjunction of two sentences and similar operations can be expressed in the language of arithmetic. Since the language of arithmetic does not contain any function symbol apart from the symbol for successor, these operations must be expressed by sutiable predicate expressions. Since the language of arithmetic does not contain a function symbol representing the function that sends sentences to their negations, appropriate paraphrases involving predicates must be given. In general, the quantified versions are stronger than the corresponding schemata. Predicates suitable as truth predicates for sublanguages of the language of arithmetic can be defined within the language of arithmetic, as long as the quantificational complexity of the formulas in the sublanguage is restricted. The definable truth predicates are truly redundant, because they are expressible in PA; therefore there is no need to introduce them axiomatically. All truth predicates in the following are not definable in the language of arithmetic, and therefore not redundant at least in the sense that they are not definable. This may be shown by appealing to the Compactness Theorem. Underlining the variable indicates it is bound from the outside. Axioms 2â€"6 claim that truth commutes with all connectives and quantifiers. Axiom 5 says that a universally quantified sentence of the language of arithmetic is true if and only if all its numerical instances are true. If these axioms are to be formulated for a language like set theory that lacks names for all objects, then axioms 5 and 6 require the use of a satisfaction relation rather than a unary truth predicate. This follows from a result due to Lachlan However, this result is sensitive to the choice of the base theory: This theory is no longer conservative over its base theory PA. For instance one can formalise the soundness theorem or global reflection principle for PA, that is, the claim that all sentences provable in PA are true. T PA is much stronger than the mere consistency statement for PA: ACA is given by the axioms of PA with full induction in the second-order language and the following comprehension principle: In T PA , quantification over sets can be defined as quantification over formulas with one free variable and membership as the truth of the formula as applied to a number. In fact, this theory proves neither the statement that all logical validities are true global reflection for pure first-order logic nor that all the Peano axioms of arithmetic are true. Perhaps surprisingly, of these two unprovable statements it is the former that is the stronger. The latter can be added as an axiom and the theory remains conservative over PA Enayat and Visser , Leigh Uniform reflection exactly captures the difference between the two theories: One adds to the language of PA truth predicates indexed by ordinals or ordinal notations or one adds a binary truth predicate that applies to ordinal notations and sentences. In this respect the hierarchical approach does not fit the framework outlined in Section 2 , because the language does not feature a single unary truth predicate applying to sentences but rather many unary truth predicates or a single binary truth predicate or even a single unary truth predicate applying to pairs of ordinal notations and sentences. On the proof-theoretic side iterating truth theories in the style of T PA corresponds to iterating elementary comprehension, that is, to iterating ACA. The system of iterated truth theories corresponds to the system of ramified analysis see Feferman Visser has studied non-wellfounded hierarchies of languages and axiomatizations thereof. Type-free truth The truth predicates in natural languages do not come with any ouvert type restriction. Therefore typed theories of truth axiomatic as well as semantic theories have been thought to be inadequate for analysing the truth predicate of natural language, although recently hierarchical theories have been advocated by Glanzberg forthcoming and others. This is one motive for investigating type-free theories of truth, that is, systems of truth that allow one to prove the truth of sentences involving the truth predicate.

Chapter 2 : Set Theory - Books Sitemap

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There are two theories that may be put forward. Several theories have been propounded by ethnologists. Even at this stage the vindictive or retributive character of punishment remains, but gradually, and specially after the humanist movement under thinkers like Beccaria and Jeremy Bentham, new theories begin to emerge. The distinguished after writers, whom we have to regard as repeating in essence pre-Kantian theories, generally know Kant, and frequently show traces of him in detail. The innumerable theories which were framed as to the precise nature of the offering and as to the precise change in the elements all implied that conception of it. In a decree of the parlement against Cartesian and other unlicensed theories was on the point of being issued, and was only checked in[time by the appearance of a burlesque mandamus against the intruder Reason, composed by Boileau and some of his brother-poets. The limited knowledge which we possess of the original features of the ground within the area of the city makes a reconstruction of the topographical history of the latter a difficult task; and, as a natural result, many irreconcilable theories have been suggested. Many theories of the relation of human to animal sacrifice have been put forward, most of them on an insufficient basis of facts. Fortunately the Cartesian method had already done its service, even where the theories were rejected. Apart from the weighty arguments which the development furnishes against the theories of Allman and Mechnikov, it may be pointed out that neither hypothesis gives a satisfactory explanation of a structure universally present in medusae of whatever class, namely the endoderm-lamella, discovered by the brothers O. Inasmuch as Lamarck attempted to frame a theory of evolution in which the principle of natural selection had no part, the interpretation placed on their work by many bionomical investigators recalls the theories of Lamarck, and the name Neo-Lamarckism has been used of such a school of biologists, particularly active in America. The old tendency illustrated by the outcome of the revolutionary movements of was once more in evidence - the tendency of merely artificial theories of democratic liberty to succumb to the immemorial instinct of race and race ascendancy. On this point the following theories have been put forward. Good cytological evidence has been adduced in favor of both theories, but further investigation is necessary before any definite conclusion can be arrived at. Useful and suggestive as they often are, teratological facts played, at one time, too large a part in the framing of morphological theories; for it was thought that the monstrous form gave a clue to the essential nature of the organ assuming it. For some of these we have no certain information, and regarding others the tales narrated in the early records are so hard to reconcile with present knowledge that they are better fitted to be the battle-ground of scholars championing rival theories than the basis of definite history. This gave immense vogue to wider and vaguer theories of evolutionary process, notably to H. But when this happened, Cartesianism was no longer either interesting or dangerous; its theories, taught as ascertained and verified truths, were as worthless as the systematic verbiage which preceded them. Flint has dealt with the following antitheistic theories: The general theories of Siphonophoran morphology are discussed below, but in enumerating the various types of appendages it is convenient to discuss their morphological interpretation, at the same time. The difference between the theories of Haeckel and Chun is connected with a further divergence in the interpretation of the stem or axis of the cormus. The many theories that have been put forward as to the interpretation of the cormus and the various parts are set forth and discussed in the treatise of Y. Passing from mythology to speculation properly so called, we find in the early systems of philosophy of India theories of emanation which approach in some respects the idea of evolution. In some of these we see a return to Greek theories, though the influence of physical discoveries, more especially those of Copernicus, Kepler and Galileo, is distinctly traceable. These theories, however, contain little that bears directly on the hypothesis of a natural evolution of things. Almost every side of zoology has contributed to the theory of evolution, but of special importance are the facts and theories associated with the names of Gregor Mendel, A.

There are two main theories: I that the chromosomes which finally separate are at first paired side by side Allen, Grgoire, Berghs, Strasburger and others , and 2 that they are joined together or paired end to end Farmer and Moore, Gregory, Mottier and others. In he published his treatise Les Beaux-Arts adults a un meme Principe, an attempt to find a unity among the various theories of beauty and taste, and his views were widely accepted. Investigations of every kind which have been based on original sources of knowledge may be styled "research," and it may be said that without "research" no authoritative works have been written, no scientific discoveries or inventions made, no theories of any value propounded; but the word also has a somewhat restricted meaning attached to it in current usage. Dealing less with theories than with facts, and illustrating rather than formdating the principles of the science. In fact, he belonged to the old Ionian school, whose doctrines he modified by the theories of his contemporary Anaxagoras, although he avoided his dualism. The address in reply to the speech from the throne, voted after a debate in which abstract theories had triumphed over common sense, demanded universal suffrage, the establishment of pure parliamentary government, the abolition of capital punishment, the expropriation of the landlords, a political amnesty, and the suppression of the Imperial Council. These were, in fact, simply the popular theories of sacrifice put on an evidential basis by facts drawn from various stages of culture. In a world without scarcity, or that has scarcity at such a trivial level it is hardly noticeable, all the conventional theories and dogmas lose their meaning. I do remember some theories concerning relativity suggesting some sort of motion in space might allow time travel if space-time geometrics are possible. Let us consider some common phenomena in the light of these rival theories as to the nature of matter. Especially he had written to Pere Mesland, one of the order, to show how the Catholic doctrine of the eucharist might be made compatible with his theories of matter. The purely philosophical theories of Aquinas are explained in the article Scholasticism. Given so many different nutritional theories and viewpoints, most people base their own nutritional philosophies on a combination of two factors: Miss Keller is distinctly not a singular proof of occult and mysterious theories, and any attempt to explain her in that way fails to reckon with her normality. The points in relation to this offering which are clearly demonstrable from the Christian writers of the first two centuries, but which subsequent theories have tended to confuse, are these. To enter here into an exhaustive account of the various theories which even before, though especially after, the appearance of the Constitution of Athens have been propounded as to the chronology of the Peisistratean tyranny, is impossible. THE] were practically powerless, the more so as their political activity consisted mainly in " building theories for an imaginary world.

Chapter 3 : mathematical philosophy - When is a statement provable? - MathOverflow

Title / Author Type Language Date / Edition Publication; 1. On sentences provable in impredicative extensions of theories: 1.

Simpson 1 First draft: March 21, This draft: Compatibility of the Arithmetical Axioms. Determination of the Solvability of a Diophantine Equation. Expression of Definite Forms by Squares. In a later paper [33] published in , Hilbert further elaborated his ideas on the importance of consistency proofs. Let be any theory in the predicate calculus satisfying certain well-known mild conditions. Then we have the following results: Some commentators have asserted that the Incompleteness Theorems mark the end of the axiomatic method. However, I would argue that this view fails to take account of developments in the foundations of mathematics subsequent to The purpose of this paper is to call attention to some relatively recent research which reveals a large amount of logical regularity and structure arising from the Incompleteness Theorems and from the axiomatic approach to foundations of mathematics. Foundational consequences of Reverse Mathematics. Using the Second Incompleteness Theorem as our jumping-off point, we define an ordering of theories as follows. Let and be two theories as above. One sometimes says that the consistency strength of is less than that of. Often this goes hand-in-hand with saying that is interpretable in and not vice versa. It is known that the ordering gives rise to a hierarchy of foundationally significant theories, ordered by consistency strength. Each of the theories in Table 1 is of considerable significance for the foundations of mathematics. Generally speaking, the idea of Table 1 is that the lower theories are below the higher theories with respect to the ordering. The exception is that , , and are all of the same consistency strength. A number of these theories will be described below in connection with Reverse Mathematics. It is striking that a great many foundational theories are linearly ordered by. Of course it is possible to construct pairs of artificial theories which are incomparable under. The problem of explaining this observed regularity is a challenge for future foundational research. As an alternative to the ordering, one may consider a somewhat different ordering, the inclusion ordering. Our jumping-off point here is the First Incompleteness Theorem. Assuming that the language of is part of the language of , let us write to mean that the sentences which are theorems of form a proper subset of the sentences in the language of which are theorems of.

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Chapter 4 : The GÃ¶del Hierarchy and Reverse Mathematics

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The sequent calculi Each of these can give a complete and axiomatic formalization of propositional or predicate logic of either the classical or intuitionistic flavour, almost any modal logic , and many substructural logics , such as relevance logic or linear logic. Indeed, it is unusual to find a logic that resists being represented in one of these calculi. Proof theorists are typically interested in proof calculi that support a notion of analytic proof. The notion of analytic proof was introduced by Gentzen for the sequent calculus; there the analytic proofs are those that are cut-free. Much of the interest in cut-free proofs comes from the subformula property: This allows one to show consistency of the sequent calculus easily; if the empty sequent were derivable it would have to be a subformula of some premise, which it is not. The definition is slightly more complex: Structural proof theory is connected to type theory by means of the Curry-Howard correspondence , which observes a structural analogy between the process of normalisation in the natural deduction calculus and beta reduction in the typed lambda calculus. Other research topics in structural theory include analytic tableau, which apply the central idea of analytic proof from structural proof theory to provide decision procedures and semi-decision procedures for a wide range of logics, and the proof theory of substructural logics. Ordinal analysis Ordinal analysis is a powerful technique for providing combinatorial consistency proofs for subsystems of arithmetic, analysis, and set theory. Ordinal analysis allows one to measure precisely the infinitary content of the consistency of theories. For a consistent recursively axiomatized theory T, one can prove in finitistic arithmetic that the well-foundedness of a certain transfinite ordinal implies the consistency of T. Consequences of ordinal analysis include 1 consistency of subsystems of classical second order arithmetic and set theory relative to constructive theories, 2 combinatorial independence results, and 3 classifications of provably total recursive functions and provably well-founded ordinals. Ordinal analysis has been extended to many fragments of first and second order arithmetic and set theory. One major challenge has been the ordinal analysis of impredicative theories. The point is to capture the notion of a proof predicate of a reasonably rich formal theory. Some of the basic results concerning the incompleteness of Peano Arithmetic and related theories have analogues in provability logic. There are also modal analogues of the fixed-point theorem. That is, the propositional theory of provability in Peano Arithmetic is completely represented by the modal logic GL. This straightforwardly implies that propositional reasoning about provability in Peano Arithmetic is complete and decidable. Other research in provability logic has focused on first-order provability logic, polymodal provability logic with one modality representing provability in the object theory and another representing provability in the meta-theory , and interpretability logics intended to capture the interaction between provability and interpretability. Some very recent research has involved applications of graded provability algebras to the ordinal analysis of arithmetical theories. Reverse mathematics Reverse mathematics is a program in mathematical logic that seeks to determine which axioms are required to prove theorems of mathematics. Its defining method can be described as "going backwards from the theorems to the axioms ", in contrast to the ordinary mathematical practice of deriving theorems from axioms. The goal of reverse mathematics, however, is to study possible axioms of ordinary theorems of mathematics rather than possible axioms for set theory. In reverse mathematics, one starts with a framework language and a base theoryâ€"a core axiom systemâ€"that is too weak to prove most of the theorems one might be interested in, but still powerful enough to develop the definitions necessary to state these theorems. For example, to study the theorem "Every bounded sequence of real numbers has a supremum " it is necessary to use a base system that can speak of real numbers and sequences of real numbers. For each theorem that can be stated in the base system but is not provable in the base system, the goal is to determine the particular axiom system stronger than the base system that is necessary to prove that theorem. To show that a system S is required to prove a theorem T, two proofs are required. The first proof shows T is provable from S; this is an ordinary mathematical proof along with a justification that it can be carried out in the system S. The second proof, known as a reversal, shows that T itself implies S; this proof is carried out in the base system. One striking phenomenon in reverse mathematics is the robustness of the Big Five axiom systems. Nearly every theorem of ordinary mathematics that has been reverse mathematically analyzed has been proven equivalent to one of these five systems. Research in reverse mathematics often incorporates methods and techniques from recursion theory as well as proof theory. Functional interpretations[edit] Functional interpretations are interpretations of non-constructive theories in functional ones. Functional interpretations usually proceed in two stages. First, one "reduces" a classical theory C to an intuitionistic one I. That is, one provides a constructive mapping that translates the theorems of C to the theorems of I. Second, one reduces the intuitionistic theory I to a quantifier free theory of functionals F. Successful functional interpretations have yielded reductions of infinitary theories to finitary theories and impredicative theories to predicative ones. Functional interpretations also provide a way to extract constructive information from proofs in the reduced theory. As a direct consequence of the interpretation one usually obtains the result that any recursive function whose totality can be proven either in I or in C is represented by a term of F. If one can provide an additional interpretation of F in I, which is sometimes possible, this characterization is in fact usually shown to be exact. It often turns out that the terms of F coincide with a natural class of functions, such as the primitive recursive or polynomial-time computable functions. Functional interpretations have also been used to provide ordinal analyses of theories and classify their provably recursive functions. This interpretation is commonly known as the Dialectica interpretation. Together with the double-negation interpretation of classical logic in intuitionistic logic, it provides a reduction of classical arithmetic to intuitionistic arithmetic. Formal and informal proof[edit] Main article: Formal proof The informal proofs of everyday mathematical practice are unlike the formal proofs of proof theory. They are rather like high-level sketches that would allow an expert to reconstruct a formal proof at least in principle, given enough time and patience. For most mathematicians, writing a fully formal proof is too pedantic and long-winded to be in common use. Formal proofs are constructed with the help of computers in interactive theorem proving. Significantly, these proofs can be checked automatically, also by computer. Checking formal proofs is usually simple, whereas finding proofs automated theorem proving is generally hard. An informal proof in the mathematics literature, by contrast, requires weeks of peer review to be checked, and may still contain errors.

Chapter 5 : On sentences provable in impredicative extensions of theories (Book,) [theinnatdunvilla.com]

The set and set existence constitute an important part of the research in mathematical logic and the foundations of mathematics. Zermelo-Fraenkel set theory with or without the axiom of choice is the generally accepted formalism and can be considered as the official framework for doing mathematics.

Ordinal systems part 2: Further we could give another example so called st-connectivity for the fact that tree-like resolution does not polynomially simulate ordinary resolution. On PHP st-connectivity, and odd charged graphs. Proof complexity and feasible arithemetics. American Mathematical Society, , pp. This is done by defining an extensional equality explicitly. Springer Lecture Notes in Computer Science, , , pp. This allows as well, to prove cut elimination for certain fragments of set theory. A solution for this problem can be found in A. Translating set theoretical proofs into type theoretical programs. Computational Logic and Proof Theory. There is a different and very elegant approach by Buchholz. Free Algebras for Iterated Inductive Definitions. As part of the research of the group in Munich on proof assistants I have introduced a type theory without dependent types having the strength of finitely iterated inductive definitions. This was an extension of free algebras studied in computer science by allowing infinite branching. I have analysed it proof theoretically by giving an extremely simple well ordering proof. This allows the use of the machinery implemented in the theorem prover MINLOG in Munich for extracting programs from classical proofs for these theories with full extensionality. We are working on proofs for this. It was difficult to get an intuitive understanding of ordinal systems for this strength. This proof is relative intuitive. However, we have some approaches which seem to cross that border and are investigating, whether they will lead to extensions which go substantially byond it. Unfortunately the literature in this area is not very well developed yet Drafts by Toshiyasu Arai , Hiroshima and Michael Rathjen , Leeds.

Chapter 6 : Use theories in a sentence | theories sentence examples

Ratajczyk, Z. [] On sentences provable in impredicative extensions of theories, Dissertationes Mathematicae CLXXVIII, pp Google Scholar Vaught, R.L. [] Axiomatizability by a schema, J. Symbolic Logic 32,pp.

One important theorem, the normalisation theorem, says that this cannot happen with simple types: The need for such a hierarchy is hinted in Appendix B of Russell Interestingly this is connected there to the question of the identity of equivalent propositions and of the logical product of a class of propositions. A thorough discussion can be found in Chapter 10 of Russell Since this notion of ramified hierarchy has been extremely influential in logic and especially proof theory, we describe it in some details. In order to further motivate this hierarchy, here is one example due to Russell. If we say Napoleon was Corsican. If we say on the other hand Napoleon had all the qualities of a great general we are referring to a totality of qualities. Another example, also coming from Russell, shows how impredicative properties can potentially lead to problems reminiscent of the liar paradox. Suppose that we suggest the definition A typical Englishman is one who possesses all the properties possessed by a majority of Englishmen. It is clear that most Englishmen do not possess all the properties that most Englishmen possess. Therefore, a typical Englishman, according to this definition, should be untypical. It is remarkable that similar problems arise when defining the notion of random numbers, cf. Russell introduced the ramified hierarchy in order to deal with the apparent circularity of such impredicative definitions. One should make a distinction between the first-order properties, like being Corsican, that do not refer to the totality of properties, and consider that the second-order properties refer only to the totality of first-order properties. One can then introduce third-order properties, that can refer to the totality of second-order property, and so on. This clearly eliminates all circularities connected to impredicative definitions. Instead, one should introduce a ramified hierarchy of properties and numbers. At the beginning, one has only first-order inductive properties, which do not refer in their definitions to a totality of properties, and one defines the numbers of order 1 to be the elements satisfying all first-order inductive properties. One can next consider the second-order inductive properties, that can refer to the collection of first-order properties, and the numbers of order 2, that are the elements satisfying the inductive properties of order 2. One can then similarly consider numbers of order 3, and so on. We have thus a sequence of more and more restricted properties: This example illustrates well the complexities introduced by the ramified hierarchy. The complexities are further amplified if one, like Russell as for Frege, defines even basic objects like natural numbers as classes of classes. For instance the number 2 is defined as the class of all classes of individuals having exactly two elements. We again obtain natural numbers of different orders in the ramified hierarchy. Besides Russell himself, and despite all these complications, Chwistek tried to develop arithmetic in a ramified way, and the interest of such an analysis was stressed by Skolem. See Burgess and Hazen for a recent development. Another mathematical example, often given, of an impredicative definition is the definition of least upper bound of a bounded class of real numbers. If we identify a real with the set of rationals that are less than this real, we see that this least upper bound can be defined as the union of all elements in this class. Let us identify subsets of the rationals as predicates. One obtains then not one notion or real numbers, but real numbers of different orders 1, 2, â€¦ The least upper bound of a collection of reals of order 1 will then be at least of order 2 in general. How should one deal with the complications introduced by the ramified hierarchy? Russell showed, in the introduction to the second edition to Principia Mathematica , that these complications can be avoided in some cases. He even thought, in Appendix B of the second edition of Principia Mathematica, that the ramified hierarchy of natural numbers of order 1,2,â€¦ collapses at order 5. Can this be done in a ramified hierarchy? Russell doubted that this could be done within a ramified hierarchy of predicates and this was indeed confirmed indeed later Heck Because of these problems, Russell and Whitehead introduced in the first edition of Principia Mathematica the following reducibility axiom: This means that for any predicate of any order, there is a predicate of the first-order level which is equivalent to it. The motivation for this axiom was purely pragmatic. Without it, all basic mathematical notions, like real or natural numbers are stratified into different orders. Also, despite the apparent circularity of impredicative definitions, the axiom of reducibility does not seem to lead to inconsistencies. As noticed however first by Chwistek, and later by Ramsey, in the presence of the axiom of reducibility, there is actually no point in introducing the ramified hierarchy at all! It is much simpler to accept impredicative definitions from the start. The axiom of reducibility draws attention to the problematic status of impredicative definitions. So far, no contradictions have been found using the reducibility axiom. However, as we shall see below, proof-theoretic investigations confirm the extreme strength of such a principle. The idea of the ramified hierarchy has been extremely important in mathematical logic. Russell considered only the finite iteration of the hierarchy: This shows that the introduction of transfinite orders can in some case play the role of the axiom of reducibility. This model satisfies in this way the Continuum Hypothesis, and gives a relative consistency proof of this axiom. The ramified hierarchy has been also the source of much work in proof theory. Such an ordinal was obtained independently about the same time by S. Recent proof-theoretical works however are concerned with systems having bigger proof-theoretical ordinals that can be considered predicative see for instance Palmgren Besides these proof theoretic investigations related to the ramified hierarchy, much work has been devoted in proof theory to analysing the consistency of the axiom of reducibility, or, equivalently, the consistency of impredicative definitions. This conjecture seemed at first extremely dubious given the circularity of impredicative quantification, which is well reflected in this formalism. One shows that this implies the consistency of a suitable form of infinity axiom, see Andrews Prawitz that indeed the cut-elimination property holds the proof of this has to use a stronger proof theoretic principle, as it should be according to the incompleteness theorem. What is important here is that these studies have revealed the extreme power of impredicative quantification or, equivalently, the extreme power of the axiom of reducibility. Another research direction in proof theory has been to understand how much of impredicative quantification can be explained from principles that are available in intuitionistic mathematics. The strongest such principles are strong forms of inductive definitions. Interestingly, almost all known uses of impredicative quantifications: Leibniz equality, least upper bound, etc. It can be seen as a constructive explanation of some restricted, but nontrivial, uses of impredicative definitions. It is clear intuitively how we can explain type theory in set theory: The other direction is more interesting. How can we explain the notion of sets in terms of types? There is an elegant solution, due to A. Miquel, which complements previous works by P. Aczel and which has also the advantage of explaining non necessarily well-founded sets a la Finsler. One simply interprets a set as a pointed graph where the arrow in the graph represents the membership relation. We can then define the membership relation: It can then be checked that all the usual axioms of set theory extensionality, power set, union, comprehension over bounded formulae and even antifoundation, so that the membership relation does not need to be well-founded hold in this simple model. We limit ourselves to presenting two applications of type theory to category theory: The terms are extended by adding pairing operations and projections and a special element of type 1. As in Lambek and Scott , one can then define a notion of typed conversions between terms, and show that this relation is decidable. This can be used to show that equality between arrows in this category is decidable. The theory of types of Church can also be used to build the free topos. It should be noted that the type theory in Lambek and Scott uses a variation of type theory, introduced by Henkin and refined by P. Andrews which is to have an extensional equality as the only logical connective, i. In set theory, the powerset operation can be iterated transfinitely along the cumulative hierarchy. It is then natural to look for analogous transfinite versions of type theory. Adding a universe is a reflection process: More powerful form of universes are considered in Palmgren Miquel presents a version of type theory of strength equivalent to the one of Zermelo-Fraenkel. This was suggested by P. Girard showed that the resulting type theory is inconsistent as a logical system Girard Girard first obtained his paradox for a weaker system. This paradox was refined later Coquand and Hurkens The notion of pure type system, introduced in Barendregt , is convenient for getting a sharp formulation of these paradoxes. Notice the circularity, indeed of the same kind as the one that is rejected by the ramified hierarchy: Despite this circularity, J. Girard was able to show normalisation for type systems with this form of polymorphism. It is quite remarkable that the circularity inherent in impredicativity does not result in non-normalisable terms. A similar system was introduced independently by J. Reynolds while analysing the notion of polymorphism in computer science. It is worth recalling here his three motivating points: The alternative choice of taking away 3 is discussed in Coquand Univalent Foundations The connections between type theory, set theory and category theory gets a new light through the work on Univalent Foundations Voevodsky and the Axiom of Univalence.

Chapter 7 : theinnatdunvilla.com - Show that Z2 is not conservative over PA - MathOverflow

Zygmunt Ratajczyk was a deep and subtle mathematician who, with mastery, used sophisticated and technically complex methods, in particular combinatorial and proof-theoretic ones. Walking always along his own paths and being immune from actual trends and fashions he hesitated to publish his results, looking endlessly for their improvement.

General principles[edit] In reverse mathematics, one starts with a framework language and a base theoryâ€"a core axiom systemâ€"that is too weak to prove most of the theorems one might be interested in, but still powerful enough to develop the definitions necessary to state these theorems. For each theorem that can be stated in the base system but is not provable in the base system, the goal is to determine the particular axiom system stronger than the base system that is necessary to prove that theorem. To show that a system S is required to prove a theorem T, two proofs are required. The first proof shows T is provable from S; this is an ordinary mathematical proof along with a justification that it can be carried out in the system S. The second proof, known as a reversal, shows that T itself implies S; this proof is carried out in the base system. Use of second-order arithmetic[edit] Most reverse mathematics research focuses on subsystems of second-order arithmetic. The body of research in reverse mathematics has established that weak subsystems of second-order arithmetic suffice to formalize almost all undergraduate-level mathematics. In second-order arithmetic, all objects can be represented as either natural numbers or sets of natural numbers. For example, in order to prove theorems about real numbers, the real numbers can be represented as Cauchy sequences of rational numbers , each of which can be represented as a set of natural numbers. The axiom systems most often considered in reverse mathematics are defined using axiom schemes called comprehension schemes. Such a scheme states that any set of natural numbers definable by a formula of a given complexity exists. In this context, the complexity of formulas is measured using the arithmetical hierarchy and analytical hierarchy. The reason that reverse mathematics is not carried out using set theory as a base system is that the language of set theory is too expressive. Extremely complex sets of natural numbers can be defined by simple formulas in the language of set theory which can quantify over arbitrary sets. Another effect of using second-order arithmetic is the need to restrict general mathematical theorems to forms that can be expressed within arithmetic. For example, second-order arithmetic can express the principle "Every countable vector space has a basis" but it cannot express the principle "Every vector space has a basis". In practical terms, this means that theorems of algebra and combinatorics are restricted to countable structures, while theorems of analysis and topology are restricted to separable spaces. Many principles that imply the axiom of choice in their general form such as "Every vector space has a basis" become provable in weak subsystems of second-order arithmetic when they are restricted. For example, "every field has an algebraic closure" is not provable in ZF set theory, but the restricted form "every countable field has an algebraic closure" is provable in RCA0, the weakest system typically employed in reverse mathematics. Use of higher-order arithmetic[edit] A recent strand of higher-order reverse mathematics research, initiated by Ulrich Kohlenbach , focuses on subsystems of higher-order arithmetic Kohlenbach Higher-order reverse mathematics includes higher-order versions of second-order comprehension schemes. Such an higher-order axiom states the existence of a functional that decides the truth or falsity of formulas of a given complexity. In this context, the complexity of formulas is also measured using the arithmetical hierarchy and analytical hierarchy. The higher-order counterparts of the major subsystems of second-order arithmetic generally prove the same second-order sentences or a large subset as the original second-order systems see Kohlenbach and Hunter As noted in the previous paragraph, second-order comprehension axioms easily generalize to the higher-order framework. However, theorems expressing the compactness of basic spaces behave quite differently in second- and higher-order arithmetic: Other covering lemmas e. The big five subsystems of second-order arithmetic[edit] Second-order arithmetic is a formal theory of the natural numbers and sets of natural numbers. Many mathematical objects, such as countable rings , groups , and fields , as well as points in effective Polish spaces , can be represented as sets of natural numbers, and modulo this representation can be studied in second-order arithmetic. Reverse mathematics makes use of several subsystems of second-order arithmetic. A typical reverse mathematics theorem shows that a particular mathematical theorem T is equivalent to a particular subsystem S of second-order arithmetic over a weaker subsystem B. This weaker system B is known as the base system for the result; in order for the reverse mathematics result to have meaning, this system must not itself be able to prove the mathematical theorem T. The following table summarizes the "big five" systems Simpson , p.

Chapter 8 : Axiomatic Theories of Truth (Stanford Encyclopedia of Philosophy)

An axiomatic theory of truth is a deductive theory of truth as a primitive undefined predicate. Because of the liar and other paradoxes, the axioms and rules have to be chosen carefully in order to avoid inconsistency.

One important theorem, the normalisation theorem, says that this cannot happen with simple types: A then M is normalisable in a strong way any sequence of reductions starting from M terminates. The need for such a hierarchy is hinted in Appendix B of Russell Interestingly this is connected there to the question of the identity of equivalent propositions and of the logical product of a class of propositions. A thorough discussion can be found in Chapter 10 of Russell Since this notion of ramified hierarchy has been extremely influential in logic and especially proof theory, we describe it in some details. In order to further motivate this hierarchy, here is one example due to Russell. If we say Napoleon was Corsican. If we say on the other hand Napoleon had all the qualities of a great general we are referring to a totality of qualities. Another example, also coming from Russell, shows how impredicative properties can potentially lead to problems reminiscent of the liar paradox. Suppose that we suggest the definition A typical Englishman is one who possesses all the properties possessed by a majority of Englishmen. It is clear that most Englishmen do not possess all the properties that most Englishmen possess. Therefore, a typical Englishman, according to this definition, should be untypical. It is remarkable that similar problems arise when defining the notion of random numbers, cf. Russell introduced the ramified hierarchy in order to deal with the apparent circularity of such impredicative definitions. One should make a distinction between the first-order properties, like being Corsican, that do not refer to the totality of properties, and consider that the second-order properties refer only to the totality of first-order properties. One can then introduce third-order properties, that can refer to the totality of second-order property, and so on. This clearly eliminates all circularities connected to impredicative definitions. Instead, one should introduce a ramified hierarchy of properties and numbers. At the beginning, one has only first-order inductive properties, which do not refer in their definitions to a totality of properties, and one defines the numbers of order 1 to be the elements satisfying all first-order inductive properties. One can next consider the second-order inductive properties, that can refer to the collection of first-order properties, and the numbers of order 2, that are the elements satisfying the inductive properties of order 2. One can then similarly consider numbers of order 3, and so on. We have thus a sequence of more and more restricted properties: This example illustrates well the complexities introduced by the ramified hierarchy. The complexities are further amplified if one, like Russell as for Frege, defines even basic objects like natural numbers as classes of classes. For instance the number 2 is defined as the class of all classes of individuals having exactly two elements. We again obtain natural numbers of different orders in the ramified hierarchy. Besides Russell himself, and despite all these complications, Chwistek tried to develop arithmetic in a ramified way, and the interest of such an analysis was stressed by Skolem. See Burgess and Hazen for a recent development. Another mathematical example, often given, of an impredicative definition is the definition of least upper bound of a bounded class of real numbers. If we identify a real with the set of rationals that are less than this real, we see that this least upper bound can be defined as the union of all elements in this class. Let us identify subsets of the rationals as predicates. For example, for rational numbers q, P q holds iff q is a member of the subset identified as P. Now, we define the predicate LC a subset of the rationals to be the least upper bound of class C as: In the ramified hierarchy, if C is a class of first-order classes of rationals, then L will be a second-order class of rationals. One obtains then not one notion or real numbers, but real numbers of different orders 1, 2, â€¦ The least upper bound of a collection of reals of order 1 will then be at least of order 2 in general. How should one deal with the complications introduced by the ramified hierarchy? Russell showed, in the introduction to the second edition to Principia Mathematica , that these complications can be avoided in some cases. He even thought, in Appendix B of the second edition of Principia Mathematica, that the ramified hierarchy of natural numbers of order 1,2,â€¦ collapses at order 5. Can this be done in a ramified hierarchy?

Russell doubted that this could be done within a ramified hierarchy of predicates and this was indeed confirmed indeed later Heck Because of these problems, Russell and Whitehead introduced in the first edition of Principia Mathematica the following reducibility axiom: This means that for any predicate of any order, there is a predicate of the first-order level which is equivalent to it. The motivation for this axiom was purely pragmatic. Without it, all basic mathematical notions, like real or natural numbers are stratified into different orders. Also, despite the apparent circularity of impredicative definitions, the axiom of reducibility does not seem to lead to inconsistencies. As noticed however first by Chwistek, and later by Ramsey, in the presence of the axiom of reducibility, there is actually no point in introducing the ramified hierarchy at all! It is much simpler to accept impredicative definitions from the start. The axiom of reducibility draws attention to the problematic status of impredicative definitions. So far, no contradictions have been found using the reducibility axiom. However, as we shall see below, proof-theoretic investigations confirm the extreme strength of such a principle. The idea of the ramified hierarchy has been extremely important in mathematical logic. Russell considered only the finite iteration of the hierarchy: This shows that the introduction of transfinite orders can in some case play the role of the axiom of reducibility. This model satisfies in this way the Continuum Hypothesis, and gives a relative consistency proof of this axiom. The ramified hierarchy has been also the source of much work in proof theory. Each u k can be computed in terms of the so-called Veblen hierarchy: Such an ordinal was obtained independently about the same time by S. Recent proof-theoretical works however are concerned with systems having bigger proof-theoretical ordinals that can be considered predicative see for instance Palmgren Besides these proof theoretic investigations related to the ramified hierarchy, much work has been devoted in proof theory to analysing the consistency of the axiom of reducibility, or, equivalently, the consistency of impredicative definitions. This conjecture seemed at first extremely dubious given the circularity of impredicative quantification, which is well reflected in this formalism. Thus the formula A[X: One shows that this implies the consistency of a suitable form of infinity axiom, see Andrews Prawitz that indeed the cut-elimination property holds the proof of this has to use a stronger proof theoretic principle, as it should be according to the incompletness theorem. What is important here is that these studies have revealed the extreme power of impredicative quantification or, equivalently, the extreme power of the axiom of reducibility. Another research direction in proof theory has been to understand how much of impredicative quantification can be explained from principles that are available in intuitionistic mathematics. The strongest such principles are strong forms of inductive definitions. Interestingly, almost all known uses of impredicative quantifications: Leibniz equality, least upper bound, etc. It can be seen as a constructive explanation of some restricted, but nontrivial, uses of impredicative definitions. It is clear intuitively how we can explain type theory in set theory: The other direction is more interesting. How can we explain the notion of sets in term of types? There is an elegant solution, due to A. Miquel, which complements previous works by P. Aczel and which has also the advantage of explaining non necessarily well-founded sets a la Finsler. One simply interprets a set as a pointed graph where the arrow in the graph represents the membership relation. This is very conveniently represented in type theory, a pointed graph being simply given by a type A and a pair of elements a: A bisimulation is a relation T: We can then define the membership relation: It can then be checked that all the usual axioms of set theory extensionality, power set, union, comprehension over bounded formulae and even antifoundation, so that the membership relation does not need to be well-founded hold in this simple model. We limit ourselves to presenting two applications of type theory to category theory: The terms are extended by adding pairing operations and projections and a special element of type 1. As in Lambek and Scott , one can then define a notion of typed conversions between terms, and show that this relation is decidable. This can be used to show that equality between arrows in this category is decidable. The theory of types of Church can also be used to build the free topos. For this we take as objects pairs A,E with A type and E a partial equivalence relation, that is a closed term E: For the subobject classifier we take the pair o, E with E: It should be noted that the type theory in Lambek and Scott uses a variation of type theory, introduced by Henkin and refined by P. Andrews which is to have an extensional equality as the only logical connective, i. T P The equality at type o is logical equivalence. In set theory, the powerset operation can be iterated transfinitely along the cumulative hierarchy. It is then natural to look for analogous transfinite versions of type theory. Adding a universe is a reflection process: U and, furthermore, A is a type if A: We can then consider types such as A: U and an element x of type A, and outputs an element of type A. More generally if T A is a type under the assumption A: U, one can form the dependent type A:

Chapter 9 : Reverse mathematics - Wikipedia

Ratajczyk, Z., Subsystems of true arithmetic and hierarchies of functions, Annals of Pure and Applied Logic 64 () The combinatorial method coming from the study of combinatorial.

Reviewed by Solomon Feferman, Stanford University This year has been a banner one for truth or, to be more precise, for conferences and workshops on philosophical and logical theories of truth. To my knowledge there have been or will shortly be at least four such, namely at Amsterdam, Princeton, Paris and Oxford, all planned independently. If we take this as an indication of a substantial upsurge of interest and activity in recent years on theories of truth, the book under review is very much of the moment and is on its way to becoming indispensable to those interested in the subject -- philosophers as well as logicians, aspiring as well as established. For it is the best book of its kind to appear in some years. Among the former the author counts such traditional theories as those of correspondence, utility, coherence, consensus, and so on, which are concerned to define the nature of truth. I think it is more informative to make instead a contrast between philosophical and logical theories of truth, so that the mentioned traditional "substantive" theories fall under the philosophical side together with such theories as those of deflationism and Davidsonian primitivism, which do not propose a definition or rather, general explanation but are concerned rather with what purposes are served by the notion of truth. On the logical side, by contrast, we have both semantic theories in which a truth-like notion is defined in precise mathematical model-theoretic terms, and axiomatic theories in which such a notion is taken as a primitive and various of its properties are laid out precisely in more or less formal terms. Halbach subsumes semantic theories under the definitional ones, but they are of a different character from the philosophical theories. With one major exception, the present book is primarily concerned with a comparative exposition of the leading axiomatic theories of truth based on classical logic. But inter alia it is much concerned with related semantic theories and with the significance of the axiomatic theories considered for philosophical theories and especially for deflationism and its kin. Aside from collections of various sorts, most notably that of Robert L. But the only books previous to the one at hand to present a systematic comparison of axiomatic theories in general are those of Andrea Cantini, Logical Frameworks for Truth and Abstraction: The division of the text into four parts is very neat. Part I "Foundations" deals with motivations for the axiomatic approach, then with some historical background -- especially the work of Tarski -- and presents the required technical background. To some extent, the reader can pick and choose his way through all this material, while those whose primary interest will be in Part IV can take the formal results on faith, after having become acquainted with the technical preliminaries in Chs. I shall largely follow this division into the four parts in the rest of my review but, given the richness of the contents of this book, can only hit the high points. Definitional theories are supposed to explain truth in terms of more basic notions, and if successful would permit its eliminability. One of the motivations considered in Part I for axiomatizing truth instead of seeking a definition of it is that one often feels less certain about the nature of the concepts called on in such definitions than about the character of truth itself. For example, all sorts of difficulties are raised in the correspondence theory of truth as to the nature and properties of facts or states of affairs, of the relation of obtaining, and of correspondence itself. Moreover, they can be used to subsume non-definitional theories of truth such as deflationism and primitivism. In any case the issue of eliminability of truth appears in the axiomatic framework in terms of the question of conservativity of a formal system for truth over a base theory. Another reason for concentrating on the axiomatics of semantic theories such as those of Tarski and Kripke is that one thereby is able to separate the principles of truth from the usually far stronger set-theoretic machinery needed to construct the given models. Finally, axiomatic theories of truth can be compared with each other in a way that philosophical and semantic theories cannot, by means of the methods of interpretation and proof-theoretic reduction so that one can speak of one theory being stronger or weaker than another. Granted that truth is a property of certain kinds of objects, a basic foundational issue in Part I is the nature of those objects: Coming back to sentences vs. By contrast, the nature of propositions is obscure by comparison; one issue is whether or not they are structured objects. It is a common idea that sentences from the same or different languages can express the same proposition and hence have the same truth value. But what does it mean for a sentence to express a proposition? When do two sentences express the same proposition? Are all propositions expressible in some language? Finally, do all sentences in a given language express a proposition? When we settle, as the author does -- and as is customary in work on axiomatic theories of truth -- on sentences being the truth-bearers, one avoids dealing with all but the last of these difficult questions and concentrates instead in each axiomatic theory on a more precise question: Which sentences are the truth bearers? His conclusion was that a formal definition of truth would have to be made relative to any given language L whose basic predicates do not contain the truth predicate for L and are accepted to have a prior meaning in an associated metalanguage M. Then the definition of truth for sentences of such L is defined compositionally for all sentences of L, provided that M is sufficiently strong to carry out the requisite recursive definition. The base language L is that of PA, and what takes the place of the metalanguage M is the extension LT of L obtained by adjoining the unary predicate symbol T as a new basic symbol. The question then is what axioms about T are to be added? TB adds to PA all the Tarski-biconditionals for sentences A of L, and UTB expresses the corresponding biconditionals in a uniform way for formulas which may have free variables. In addition, in both theories one extends the induction scheme to apply to all formulas in LT, though restricted schemes are also considered. The basic such theory is dubbed CT, again with induction extended to all formulas of LT, though a restricted version is again considered. Take Sent x to express that x is the code of a sentence of L. More work carried out in full by a cut-elimination argument is needed to show that the restricted version of CT is a conservative extension of PA, but CT itself is not since it proves the consistency of PA. The strength of CT in full is shown to be the same as that of the second-order theory ACA of numbers and sets of numbers based on the arithmetical comprehension axiom. The major value of the book lies in Part III which is devoted to various theories that are type-free in the sense that one can consider x in T x to range over codes of sentences of LT or significant subsets of that. In ordinary parlance, T is allowed to be "self-applicable" or "self-referential". The feeling that typed systems and their hierarchical iterations are not natural is one of the reasons for considering type-free languages; certainly natural language makes no such restrictions. Another reason is that the kinds of generalizations claimed for a disquotational theory of truth by Quine and seen as the main purpose of a theory of truth by the deflationists are more directly facilitated in type-free theories. The problem is to skirt the paradoxes while achieving such ends. One way out was taken by Harvey Friedman and Michael Sheard in joint work from the late s. Friedman and Sheard investigated systematically the consistency of a number of theories in which some combination of these rules with other axioms is made. Halbach presents a version of one of these that he denotes FS. The system FS has: An easy model-theoretic argument is used to show that FS is consistent a proof-theoretic argument is also given. On the other hand, FS is arithmetically sound, i. If one starts with the empty sets of sentences on both sides, the least fixed points S1 and S2 are disjoint and there are sentences -- like that for the Liar -- that belong to neither, i. But other starting sets of sentences can lead to overlaps or "gluts". I used KF as a base to explain a notion of reflective closure of arithmetic by adding a suitable substitution rule and some further axioms. A frequent criticism of KF is that its external logic, i. That led Halbach and Leon Horsten in to develop a system PKF based on partial three-valued logic whose internal and external logic agree. In the first section of Part IV, Halbach addresses the question as to why, with the exception of PKF, he concentrated in this book entirely on theories formulated in classical logic. Interestingly, he has come to agree with me that the cost of working in weaker logics such as those of PKF is too high, since "nothing like sustained ordinary reasoning" can be carried out in them. On the other hand, Field and others have made use of hybrid logics that weaken certain classical principles to avoid the paradoxes, but in compensation make use of new operators with non-classical principles to strengthen them. These have attendant complications, but they deserve further study, especially as to their proof-theoretical strength. The remainder of Part IV contains interesting judicious discussions on the significance of the technical work in this book for disquotational theories and deflationism, reflection principles and reflective closure, and for applications to natural language. These are not easily summarized but are highly recommended to the philosophically minded reader: Under the guidance of a knowledgeable instructor, the book could well serve as a text for a two quarter or year long course and I would highly recommend it for such, especially when supplemented by additional readings to expand the perspective and fill in the historical background. The problem for an independent reader will be to keep track of the welter of theories: