

**Chapter 1 : Symbolic Logic | Math Goodies**

*Problem: Given the following statement, decide its truth value, and then decide the truth values of its inverse, converse, and contrapositive.  $p$ : If a polygon has four sides, it is a pentagon.*

Each area has a distinct focus, although many techniques and results are shared among multiple areas. The borderlines amongst these fields, and the lines separating mathematical logic and other fields of mathematics, are not always sharp. The method of forcing is employed in set theory, model theory, and recursion theory, as well as in the study of intuitionistic mathematics. The mathematical field of category theory uses many formal axiomatic methods, and includes the study of categorical logic, but category theory is not ordinarily considered a subfield of mathematical logic. Because of its applicability in diverse fields of mathematics, mathematicians including Saunders Mac Lane have proposed category theory as a foundational system for mathematics, independent of set theory. These foundations use toposes, which resemble generalized models of set theory that may employ classical or nonclassical logic. History[ edit ] Mathematical logic emerged in the mid-19th century as a subfield of mathematics, reflecting the confluence of two traditions: The first half of the 20th century saw an explosion of fundamental results, accompanied by vigorous debate over the foundations of mathematics. History of logic Theories of logic were developed in many cultures in history, including China, India, Greece and the Islamic world. In 18th-century Europe, attempts to treat the operations of formal logic in a symbolic or algebraic way had been made by philosophical mathematicians including Leibniz and Lambert, but their labors remained isolated and little known. Charles Sanders Peirce built upon the work of Boole to develop a logical system for relations and quantifiers, which he published in several papers from 1840 to 1850. Gottlob Frege presented an independent development of logic with quantifiers in his *Begriffsschrift*, published in 1879, a work generally considered as marking a turning point in the history of logic. The two-dimensional notation Frege developed was never widely adopted and is unused in contemporary texts. This work summarized and extended the work of Boole, De Morgan, and Peirce, and was a comprehensive reference to symbolic logic as it was understood at the end of the 19th century. Foundational theories[ edit ] Concerns that mathematics had not been built on a proper foundation led to the development of axiomatic systems for fundamental areas of mathematics such as arithmetic, analysis, and geometry. In logic, the term arithmetic refers to the theory of the natural numbers. Around the same time Richard Dedekind showed that the natural numbers are uniquely characterized by their induction properties. In addition to the independence of the parallel postulate, established by Nikolai Lobachevsky in *Lobachevsky*, mathematicians discovered that certain theorems taken for granted by Euclid were not in fact provable from his axioms. Among these is the theorem that a line contains at least two points, or that circles of the same radius whose centers are separated by that radius must intersect. Hilbert developed a complete set of axioms for geometry, building on previous work by Pasch. The success in axiomatizing geometry motivated Hilbert to seek complete axiomatizations of other areas of mathematics, such as the natural numbers and the real line. This would prove to be a major area of research in the first half of the 20th century. The 19th century saw great advances in the theory of real analysis, including theories of convergence of functions and Fourier series. Mathematicians such as Karl Weierstrass began to construct functions that stretched intuition, such as nowhere-differentiable continuous functions. Previous conceptions of a function as a rule for computation, or a smooth graph, were no longer adequate. Weierstrass began to advocate the arithmetization of analysis, which sought to axiomatize analysis using properties of the natural numbers. In 1872, Dedekind proposed a definition of the real numbers in terms of Dedekind cuts of rational numbers. Dedekind's definition is still employed in contemporary texts. Georg Cantor developed the fundamental concepts of infinite set theory. His early results developed the theory of cardinality and proved that the reals and the natural numbers have different cardinalities. Cantor. Over the next twenty years, Cantor developed a theory of transfinite numbers in a series of publications. Cantor believed that every set could be well-ordered, but was unable to produce a proof for this result, leaving it as an open problem in set theory. The discovery of paradoxes in informal set theory caused some to wonder whether mathematics itself is inconsistent, and to look for proofs of consistency. In 1900, Hilbert posed a famous

list of 23 problems for the next century. The first two of these were to resolve the continuum hypothesis and prove the consistency of elementary arithmetic, respectively; the tenth was to produce a method that could decide whether a multivariate polynomial equation over the integers has a solution. This problem asked for a procedure that would decide, given a formalized mathematical statement, whether the statement is true or false. Set theory and paradoxes[ edit ] Ernst Zermelo gave a proof that every set could be well-ordered, a result Georg Cantor had been unable to obtain. To achieve the proof, Zermelo introduced the axiom of choice , which drew heated debate and research among mathematicians and the pioneers of set theory. The immediate criticism of the method led Zermelo to publish a second exposition of his result, directly addressing criticisms of his proof Zermelo a. This paper led to the general acceptance of the axiom of choice in the mathematics community. Skepticism about the axiom of choice was reinforced by recently discovered paradoxes in naive set theory. Cesare Burali-Forti was the first to state a paradox: Zermelo b provided the first set of axioms for set theory. These axioms, together with the additional axiom of replacement proposed by Abraham Fraenkel , are now called Zermeloâ€™Fraenkel set theory ZF. This seminal work developed the theory of functions and cardinality in a completely formal framework of type theory , which Russell and Whitehead developed in an effort to avoid the paradoxes. Later work by Paul Cohen showed that the addition of urelements is not needed, and the axiom of choice is unprovable in ZF. Skolem realized that this theorem would apply to first-order formalizations of set theory, and that it implies any such formalization has a countable model. These results helped establish first-order logic as the dominant logic used by mathematicians. It showed the impossibility of providing a consistency proof of arithmetic within any formal theory of arithmetic. Hilbert, however, did not acknowledge the importance of the incompleteness theorem for some time. This leaves open the possibility of consistency proofs that cannot be formalized within the system they consider. Gentzen proved the consistency of arithmetic using a finitistic system together with a principle of transfinite induction. Beginnings of the other branches[ edit ] Alfred Tarski developed the basics of model theory. Beginning in , a group of prominent mathematicians collaborated under the pseudonym Nicolas Bourbaki to publish a series of encyclopedic mathematics texts. These texts, written in an austere and axiomatic style, emphasized rigorous presentation and set-theoretic foundations. Terminology coined by these texts, such as the words bijection, injection, and surjection , and the set-theoretic foundations the texts employed, were widely adopted throughout mathematics. Kleene introduced the concepts of relative computability, foreshadowed by Turing , and the arithmetical hierarchy. Kleene later generalized recursion theory to higher-order functionals. Kleene and Kreisel studied formal versions of intuitionistic mathematics, particularly in the context of proof theory. Formal logical systems [ edit ] At its core, mathematical logic deals with mathematical concepts expressed using formal logical systems. These systems, though they differ in many details, share the common property of considering only expressions in a fixed formal language. The systems of propositional logic and first-order logic are the most widely studied today, because of their applicability to foundations of mathematics and because of their desirable proof-theoretic properties. First-order logic First-order logic is a particular formal system of logic. Its syntax involves only finite expressions as well-formed formulas, while its semantics are characterized by the limitation of all quantifiers to a fixed domain of discourse. Early results from formal logic established limitations of first-order logic. This shows that it is impossible for a set of first-order axioms to characterize the natural numbers, the real numbers, or any other infinite structure up to isomorphism. As the goal of early foundational studies was to produce axiomatic theories for all parts of mathematics, this limitation was particularly stark. It shows that if a particular sentence is true in every model that satisfies a particular set of axioms, then there must be a finite deduction of the sentence from the axioms. It says that a set of sentences has a model if and only if every finite subset has a model, or in other words that an inconsistent set of formulas must have a finite inconsistent subset. The completeness and compactness theorems allow for sophisticated analysis of logical consequence in first-order logic and the development of model theory , and they are a key reason for the prominence of first-order logic in mathematics. The first incompleteness theorem states that for any consistent, effectively given defined below logical system that is capable of interpreting arithmetic, there exists a statement that is true in the sense that it holds for the natural numbers but not provable within that logical system and which indeed may fail in some non-standard models

of arithmetic which may be consistent with the logical system. Here a logical system is said to be effectively given if it is possible to decide, given any formula in the language of the system, whether the formula is an axiom, and one which can express the Peano axioms is called "sufficiently strong. Other classical logics[ edit ] Many logics besides first-order logic are studied. These include infinitary logics , which allow for formulas to provide an infinite amount of information, and higher-order logics , which include a portion of set theory directly in their semantics. The most well studied infinitary logic is  $L$ .

## Chapter 2 : Logic, Truth Values, negation, conjunction, disjunction

*Math Logic Problems* Logic puzzle problems are a set of problems which involve children using their reasoning and logical thinking skills. Sometimes children who struggle in other areas of math, such as number work, find that this is an area which they excel in.

Logical reasoning is the process of using rational, systemic steps, based on mathematical procedure, to arrive at a conclusion about a problem. You can draw conclusions based on given facts and mathematical principles. Once you master the skill in solving math problems, you can use logical reasoning in a wide array of real-world situations. Read and understand the problem. By the end of the first period, Bob had sold one-third of the hot dogs. During the second period, Bob sold 10 more hot dogs and continued selling hot dogs through the third period. When the game ended, Bob sold half the remaining grilled hot dogs. Make a plan to solve the problem backward using critical thinking and logic. In the concession stand example, you know that Bob had 10 unsold grilled hot dogs when the game ended. Working backward, start with the known quantity of 10 unsold, grilled hot dogs. You were also told that Bob sold half the remaining hot dogs when the game ended. Therefore, the second half of unsold hot dogs totals 20. Earlier, Bob had sold an additional 10 hot dogs to equal a running total of 30 hot dogs. Continuing to work backward, you recall that Bob sold one-third of his hot dogs in the first period, meaning that two-thirds remained, which equals 40. Your final calculation reveals that Bob grilled 45 hot dogs before the game started. To check the accuracy of your work, do the problem in reverse using logical reasoning. Start with your final answer of 45 hot dogs grilled before the game started. This time, however, work forward. Bob sold one-third of his hot dogs during the first period of the hockey game. Divide 45 by three, which equals 15. When you subtract 15 from 45, the answer is 30. Because Bob sold 10 more hot dogs during the second period, subtract 10 from 30, which is 20. Half of 20 is 10, which is the number of hot dogs remaining. Arriving at this solution confirms your logical reasoning abilities. He is a former commissioner with the city of Berkeley, Calif.

*Math and Logic Puzzles. If you REALLY like exercising your brain, figuring things 'round and 'round till you explode, then this is the page for you!*

This is similar to the Aristotelean syllogism, but it is of wider applicability, because the premises and the conclusion can be more complex. As an example, the 19th century logician Augustus DeMorgan noted 9 that the inference all horses are animals, therefore, the head of a horse is the head of an animal is beyond the reach of Aristotelean logic. The completeness theorem Formulas of the predicate calculus can be exceedingly complicated. How then can we distinguish the formulas that are logically valid from the formulas that are not logically valid? It turns out that there is an algorithm 10 for recognizing logically valid formulas. We shall now sketch this algorithm. In order to recognize that a formula is logically valid, it suffices to construct what is known as a proof tree for  $\phi$ , or equivalently a refutation tree for  $\neg\phi$ . This is a tree which carries at the root. Each node of the tree carries a formula. The growth of the tree is guided by the meaning of the logical operators appearing in. New nodes are added to the tree depending on what nodes have already appeared. For example, if a node carrying  $\phi \vee \psi$  has appeared, we create two new nodes carrying  $\phi$  and  $\psi$  respectively. The thought behind these new nodes is that the only way for  $\phi \vee \psi$  to be the case is if at least one of  $\phi$  or  $\psi$  is the case. Similarly, if a node carrying  $\phi \wedge \psi$  has already appeared, we create a new node carrying  $\phi$ , where  $c$  is the result of substituting a new constant for the variable. The idea here is that the only way for the universal statement to be false is if  $\phi$  is false for some particular. Since  $c$  is a new constant,  $\phi(c)$  is a formula which may be considered as the most general false instance of  $\phi$ . Corresponding to each of the seven logical operators, there are prescribed procedures for adding new nodes to the tree. We apply these procedures repeatedly until they cannot be applied any more. If explicit contradictions 11 are discovered along each and every branch of the tree, then we have a refutation tree for  $\neg\phi$ . Thus  $\phi$  is seen to be logically impossible. In other words,  $\phi$  is logically valid. The adequacy of proof trees for recognizing logically valid formulas is a major insight of 20th century logic. On the other hand, the class of logically valid formulas is known to be extremely complicated. Indeed, this class is undecidable: In this sense, the concept of logical validity is too general and too intractable to be analyzed thoroughly. There will never be a predicate calculus analog of the pons asinorum. Formal theories The predicate calculus is a very general and flexible framework for reasoning. By choosing appropriate predicates, one can reason about any subject whatsoever. These considerations lead to the notion of a formal theory. In order to specify a formal theory, one first chooses a small collection of predicates which are regarded as basic for a given field of study. These predicates are the primitives of the theory. They delimit the scope of the theory. Other predicates must be defined in terms of the primitives. Using them, one writes down certain formulas which are regarded as basic or self-evident within the given field of study. These formulas are the axioms of the theory. It is crucial to make all of our underlying assumptions explicit as axioms. Once this has been done, a theorem is any formula which is a logical consequence of the axioms. A formal theory is this structure of primitives, axioms, and theorems. As a frivolous example, we could envision a theory of cars, trucks, and drivers. The defining axioms for  $C$  and  $D$  would be  $C \wedge D$  and  $D \wedge C$ , respectively. In this fashion, we could attempt to codify all available knowledge about vehicles and drivers. More seriously, one could try to write down formal theories corresponding to various scientific disciplines, such as mechanics or statistics or law. In this way one could hope to analyze the logical structure of the respective disciplines. The process of codifying a scientific discipline by means of primitives and axioms in the predicate calculus is known as formalization. The key issue here is the choice of primitives and axioms. They cannot be chosen arbitrarily. The scientist who chooses them must exercise a certain aesthetic touch. They must be small in number; they must be basic and self-evident; and they must account for the largest possible number of other concepts and facts. To date, this kind of formal theory-building has been convincingly carried out in only a few cases. A survey is in Tarski [ 21 ]. The most notable successes have been in mathematics. Foundations of mathematics Mathematics is the science of quantity. Traditionally there were two branches of mathematics, arithmetic and geometry, dealing with two kinds of quantities: Modern mathematics is richer and deals with a wider variety of objects, but arithmetic and geometry are still of central

importance. Foundations of mathematics is the study of the most basic concepts and logical structure of mathematics, with an eye to the unity of human knowledge. Among the most basic mathematical concepts are: The reader may reasonably ask why mathematics appears at all in this volume. There are three reasons for discussing mathematics in a volume on general philosophy: Mathematics has always played a special role in scientific thought. The abstract nature of mathematical objects presents philosophical challenges that are unusual and unique. Foundations of mathematics is a subject that has always exhibited an unusually high level of technical sophistication. For this reason, many thinkers have conjectured that foundations of mathematics can serve as a model or pattern for foundations of other sciences. The philosophy of mathematics has served as a highly articulated test-bed where mathematicians and philosophers alike can explore how various general philosophical doctrines play out in a specific scientific context. The purpose of this section is to indicate the role of logic in the foundations of mathematics. We begin with a few remarks on the geometry of Euclid. We then describe some modern formal theories for mathematics. Let no one who is ignorant of geometry enter here. In this way Plato indicated his high opinion of geometry. See also Plato [ 17 , Republic, B]. In the Posterior Analytics [ 13 ], Aristotle laid down the basics of the scientific method. Euclid begins with 21 definitions, five postulates, and five common notions. After that, the rest of the Elements are an elaborate deductive structure consisting of hundreds of propositions. Each proposition is justified by its own demonstration. The demonstrations are in the form of chains of syllogisms. In each syllogism, the premises are identified as coming from among the definitions, postulates, common notions, and previously demonstrated propositions. It is true that the syllogisms of Euclid do not always conform strictly to Aristotelean templates. The logic of Aristotle and the geometry of Euclid are universally recognized as towering scientific achievements of ancient Greece. Formal theories for mathematics A formal theory for geometry With the advent of calculus in the 17th and 18th centuries, mathematics developed very rapidly and with little attention to logical foundations. But the prolific Enlightenment mathematicians such as Leonhard Euler showed almost no interest in trying to place calculus on a similarly firm foundation. Only in the last half of the 19th century did scientists begin to deal with this foundational problem in earnest. The resulting crisis had far-reaching consequences. Geometers such as Moritz Pasch discovered what they regarded as gaps or inaccuracies in the Elements. Great mathematicians such as David Hilbert entered the fray. An outcome of all this foundational activity was a thorough reworking of geometry, this time as a collection of formal theories within the predicate calculus. Decisive insights were obtained by Alfred Tarski.

## Chapter 4 : Math and Logic Problems Galore

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## Chapter 5 : K Math Problems

*Problem: Is the following sentence a statement? The car drove up the hill. Yes.*

## Chapter 6 : Logic and Mathematics

*Logical reasoning is a useful tool in many areas, including solving math problems. Logical reasoning is the process of using rational, systemic steps, based on mathematical procedure, to arrive at a conclusion about a problem.*

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## Chapter 9 : Math Logic and Math Problems

*In logic, a disjunction is a compound sentence formed by using the word or to join two simple sentences. The symbol for this is  $\hat{\vee}$ . The symbol for this is  $\hat{\vee}$ . (whenever you see  $\hat{\vee}$  read 'or') When two simple sentences,  $p$  and  $q$ , are joined in a disjunction statement, the disjunction is expressed symbolically as  $p \hat{\vee} q$ .*